Combined resonance of a nonlocal nanobeam on fractional Pasternak-type viscoelastic foundation

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Summary. In this communication, we observe the interaction of fundamental parametric resonances with subharmonic resonances of order one-half of a geometrically nonlinear nonlocal nanobeam model resting on a fractional Pasternak-type viscoelastic foundation. Euler-Bernoulli beam theory and nonlinear von Karman strain-displacement relation are employed to obtained fractional order governing equation for the transverse vibration of a system. Under the assumption of small fractional damping, we used the perturbation multiple-scales method to obtain an approximated analytical solution for the frequency-amplitude response. Combined parametric resonance from axial load and subharmonic resonance under external excitation are examined for different parameters of the model. Validation of the multiple scales solution against numerical solution in the phase plane and Poincare map will be provided.

Introduction

Nanostructures having similar shapes as macro engineering structures such as beams, plates and shells are developed using various nano-technological processes. Mechanical behavior of nano-scale structures is also similar to the behavior of macro structures with the main difference that influences of various size-effects cannot be neglected. Studying the dynamic behavior of nanostructures is especially important for the development of new types of nanoactuators, nanosensors or resonator devices. Geometrically nonlinear oscillations with different resonance conditions together with dissipation effects from different sources are important effects for the investigation and performance of nano-scale resonators. Very important is consideration of dissipation effects from external medium. Application of experimental and atomistic methods is limited for the analysis of nanostructures and it can be applied only to specific and less complex systems. Continuum based theoretical models have attracted a great attention of researchers in recent years. Eringen's nonlocal elasticity theory was one of the first that consider size effects and atomic forces through a single material parameter [1]. This theory shown to be very useful in describing the mechanical behavior of various nanostructures and nanocomposites compared to other methods [2].

Dissipation models based on fractional derivative viscoelasticity become widely used due to proven advantages compared to classical viscoelasticity models [3]. Recently, such models are successfully used to describe linear and nonlinear dynamics of nanostructures considering size and damping effects [4, 5]. When solving nonlinear dynamic problems together with the fractional derivative damping one can use numerical [5] or analytical approximation methods [6].

In this work, we will investigate a combined parametric and subharmonic resonance of a nanobeam system subjected to time dependent axial and transverse loads. Effects of fractional derivative Pasternak-type viscoelastic foundation, representing some medium around nanobeam, and size effects on frequency-amplitude response will be examined for changes of model parameters. Our intention is to fill the gap in the literature by studying the combined resonance condition of a nanobeam system using the multiple scales perturbation method to solve nonlinear fractional order differential equations. In addition, verification of the analytical perturbation method results with numerical solution will be presented in the phase plane and Poincare map.

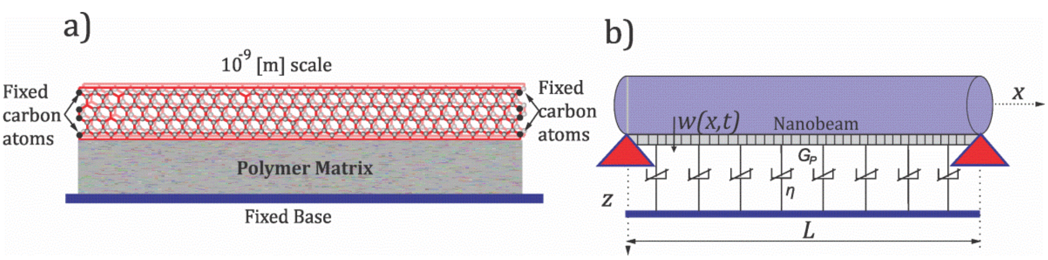

Fig.1 Illustration of a nanobeam on viscoelastic foundation a) Physical model b) Mechanical model

Problem formulation

Let as assume that a nanobeam is homogenous with length L, cross section area A, density ρ, and Young's modulus E, Fig. 1. Assuming the nonlocal elasticity stress-strain equation for one-dimensional case, von Karman strain-displacement relation and Euler-Bernoulli beam theory [6], we derive governing equation for nonlinear transverse vibration of a nanobeam resting on fractional Pasternak-type viscoelastic foundation [7] as

$$\rho A \frac{\partial^2 w}{\partial t^2} + \xi(x,t) - P\frac{\partial^2 w}{\partial x^2} - \mu\left[\rho A \frac{\partial^4 w}{\partial t^2 \partial x^2} + \frac{\partial^2 \xi(x,t)}{\partial x^2} - P\frac{\partial^4 w}{\partial x^4}\right] + EI\frac{\partial^4 w}{\partial x^4} = f(t) \quad (1)$$

where $f(t) = F_T \cos\Omega_T t$ is the assumed transverse harmonic load, $\xi(x,t) = \eta D_t^\alpha w - G_p D_t^\alpha \frac{\partial^2 w}{\partial x^2}$ is the force-displacement relation from fractional Pasternak-type foundation, where D_t^α is the operator of the Riemann-Liouville's fractional derivative. Axial force from $F = -(F_0 + F_1 \cos\Omega_A t)$ is given in the form

$$P = -(F_0 + F_1\cos\Omega_A t) + \frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \quad (2)$$

Governing equation (1) is the fractional derivative type of partial differential equation with one fractional parameter α. This equation can be solved numerically or analytically using approximation methods such as modified multiple scales perturbation technique proposed in [3]. In using this method, firstly we employ the Galerkin discretization method. In developing the multiple scales solution usual assumptions for the first and the second time derivatives are adopted while for the fractional time derivation we have

$$D_t^\alpha = \left(\frac{d}{dt}\right)^\alpha = (D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \cdots)^\alpha = D_+^\alpha + \varepsilon\alpha D_+^{\alpha-1}D_1 + \frac{1}{2}\varepsilon^2\alpha[(\alpha-1)D_+^{\alpha-2}D_1^2 + 2D_+^{\alpha-1}D_2] + \cdots \quad (3)$$

with $D_n = \frac{\partial}{\partial T_n}$ $(n = 0,1,2 \ldots)$ and D_0^α, $D_0^{\alpha-1}$, $D_0^{\alpha-2}$, … denoting the Riemann-Liouville's fractional time derivatives for $T_0 = t$, $T_1 = \varepsilon t$, $T_2 = \varepsilon^2 t$, where ε is the small parameter. Multiple scales solution of fractional order differential equation is verified with the numerical solution of the same equation. Steady state response, frequency-amplitude response and instability regions are plotted for simply supported nanobeam and different values of system parameters such as nonlocal parameter, order of fractional derivative and parameters of Pasternak-type of viscoelastic foundation. Interaction of fundamental parametric resonance from axial force with subharmonic resonance from external harmonic load is also discussed in detail.

Conclusions

This paper studies the combined fundamental parametric resonance and one half order subharmonic resonance of a nano-scale system composed of a nanobeam and fractional Pasternak-type viscoelastic foundation. Geometric nonlinearity is considered together with the Euler-Bernoulli beam theory and nonlocal elastic constitutive equation to derive motion equations of the observed system. The system is subjected to two different types of loads, transverse harmonic force and axial pulsating force. Solution of the governing equation is proposed via modified multiple scales perturbation method for fractional derivative equations, which is also confirmed with the numerical solution. Effects of different parameters on frequency-amplitude response and regions of instability are examined through several numerical examples.

Acknowledgments

This research was supported by the research grants of the Serbian Ministry of Education, Science and Technological Development under the numbers OI 174001, OI 174011, TI 35006, III 41006 and partially supported by Serbia-China bilateral project under the number 3-12.

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