Vibrations of Rotating Composite Blades with Embedded Nonlinear Piezoelectric Elements

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Summary. Dynamics of a rotating structure composed of two thin–walled blades attached to a rigid hub is presented in this paper. The box cross-section beams are made of composite material with a Circumferentially Asymmetric Stiffness (CAS) configuration. This lamination scheme results in an elastic coupling between the flapwise–bending and twisting deformations. Moreover, two active elements are symmetrically placed onto the outer surfaces of the profile flanges covering full span of the specimen. In the mathematical model of the discussed electro–mechanical system a two-way coupling piezoelectric effect is considered by adopting the assumption of a beam spanwise electric field variation. A set of governing ODEs is derived taking into account a nonconstant angular velocity and a nonlinear constitutive formula of the piezoceramic. Amplitudes of natural vibration modes and forced vibrations around the main resonance zone are presented. In contrast to the natural oscillations case, the forced vibrations exhibit softening behaviour due to the nonlinear electric field effect.

Introduction and model of a rotating structure

The development of active materials technology offers a great potential for advanced modern structural systems that are able to respond to changing operating conditions [1]. The synergistic combination of anisotropic composite material tailoring and properties of adaptive materials embedded onto the host structure are subject to intensive recent studies. In contrast to the slender elements made of isotropic materials [2, 3] thin–walled composite beams require specific treatment due to much more complex elastic behaviour and directional properties of laminates [4]. Furthermore, if additional piezoceramic layers are embedded onto the host structure a mutual coupling between mechanical and electrical domains is observed. As presented in [5] the commonly accepted linear models of PZT materials may be inaccurate in case of high power systems operated under high levels of applied voltages and/or high stresses regimes. Thus, apart from the linear mechanical couplings, also higher order terms resulting from nonlinear constitutive piezoceramic behaviour have to be taken into account.
The studied structure (Fig. 1) consists of two flexible composite beams fixed to the rigid hub that is rotating about a vertical axis. Both beams have rectangular thin-walled cross-section featuring the Circumferentially Asymmetric Stiffness (CAS) lamination scheme (see Fig. 1b). This configuration results in mutual coupling of bending and twisting elastic deformations. Moreover, there are two additional piezoceramic layers on the outer surfaces of beams profile flanges. For the studied specific case of blades spatial orientation the active PZT material is prone to deformations in the plane of rotor rotation producing quadratic and cubic electric field nonlinearities and coupled with the strain field.

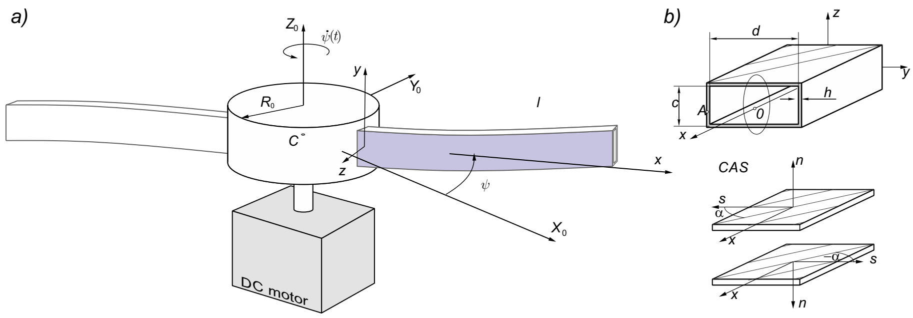

Figure 1: Model of the hub bladed rotor with two composite thin walled beam (a) and CAS composite configuration (b) .

As presented in [5] the full system of PDEs representing the dynamics of the structure can be transformed into ODEs taking into account the normal modes projection and associated orthogonality condition. The first coupled flexural–torsional mode projection results in a set of nonlinear ODEs having the dimensionless form

$$
\begin{aligned}
\ddot{q}_1 + \zeta_1\dot{q}_1 + \alpha_{12}\ddot{\psi} + (\alpha_{11} + \alpha_{13}\dot{\psi}^2)q_1 + \alpha_{14}\dot{\psi}q_1\dot{q}_1 + \alpha_{15}\,\mathrm{sgn}(q_1)q_1^2 + \alpha_{16}q_1^3 = 0 \\
\ddot{q}_2 + \zeta_2\dot{q}_2 + \alpha_{22}\ddot{\psi} + (\alpha_{21} + \alpha_{23}\dot{\psi}^2)q_2 + \alpha_{24}\dot{\psi}q_2\dot{q}_2 + \alpha_{25}\,\mathrm{sgn}(q_2)q_2^2 + \alpha_{26}q_2^3 = 0 \\
(1 + \widetilde{J}_h + \widetilde{J}_b + \alpha_{h12}q_1^2 + \alpha_{h22}q_2^2)\ddot{\psi} + \zeta_h\dot{\psi} + \alpha_{h11}\ddot{q}_1 + \alpha_{h21}\ddot{q}_2 + \alpha_{h13}\dot{\psi}q_1\dot{q}_1 + \alpha_{h23}\dot{\psi}q_2\dot{q}_2 = \mu
\end{aligned}
\tag{1}
$$

where q_1 and q_2 are the generalized coordinates corresponding to the studied coupled flexural–torsional mode of the first and the second beam, respectively. Coefficients α_{ij} $(i=1,2,h; j=1,\ldots,4)$ result from Galerkin projection, $\zeta_1, \zeta_2, \zeta_h$ are beams and the hub damping coefficients; $\widetilde{J}_h$ and $\widetilde{J}_b$ are dimensionless mass moment of inertia of the hub and the second beam, both calculated with respect to the mass inertia of the first beam. External torque supplied to the hub is assumed as a zero mean value periodic function $\mu = \rho\cos\omega t$, where ρ and ω are amplitude and frequency of excitation, respectively.

Results and conclusions

The numerical analysis is performed for the reinforcing fibers orientation angle set to $15°$ as measured with respect to spanwise axis Ox – see Fig. 1(b). The dimensionless coefficients for the studied case take values [5]: $\alpha_{11} = \alpha_{21} = 10.864, \alpha_{12} = \alpha_{22} = 1.772, \alpha_{13} = \alpha_{23} = 0.349, \alpha_{14} = \alpha_{24} = -1.55, \alpha_{15} = \alpha_{25} = -2.327, \tilde{J}_h = \tilde{J}_b = 1.0, \alpha_{16} = \alpha_{26} = 0, \alpha_{h11} = \alpha_{h21} = -0.532, \alpha_{h12} = \alpha_{h22} = -0.404, \alpha_{h13} = \alpha_{h23} = -0.808$.
Neglecting damping and external torque we may find normal modes of the structure. These are presented in the first order approximation in Fig. 2. Observed amplitudes of natural vibrations have a clear nonlinear nature, and the natural frequency of the structure depends on the oscillations amplitude – see Fig. 2(a) and (c). But as demonstrated in Fig. 2(b) and (d), corresponding modes remain linear. This solution has also been confirmed by computing the normal modes taking into account the original system (1) and applying the technique presented in [6]. Reported result is in contrast to that published in [7] for a single rotating beam made of isotropic material.

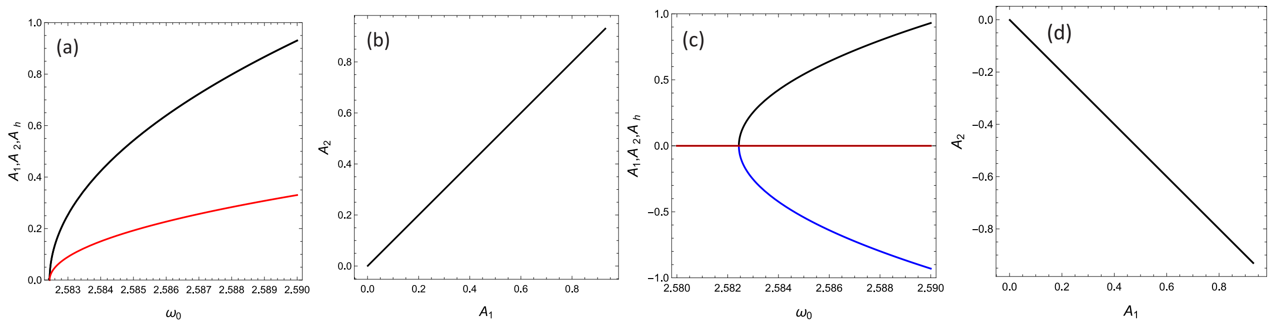

Figure 2: Amplitudes and modes of natural vibrations, (a) amplitudes A_1, A_2 (black) A_h (red) as a function of the natural frequency, (b) first vibration mode $A_1 - A_2$, (c) amplitudes A_1 (black), A_2 (blue) A_h (red), (d) second vibration mode $A_1 - A_2$.

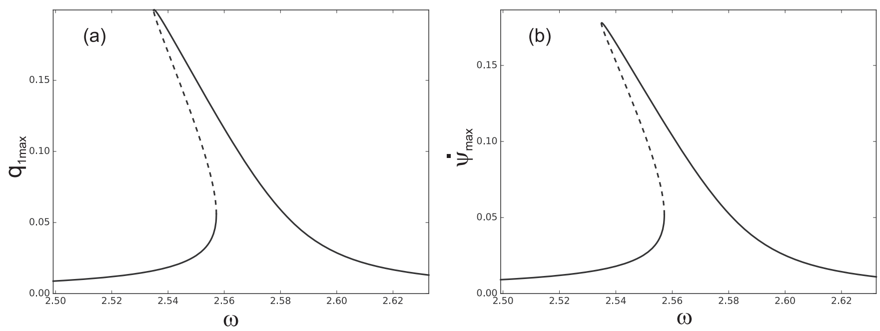

Figure 3: Resonance curves of the beam (a) and the hub (b), $\rho = 0.01$.

The influence of the nonlinear piezoelectric material properties has been tested for the forced vibration case with a zero mean value periodic torque μ supplied to the hub. Simulations have been performed for the excitation amplitude $\rho = 0.01$ and frequency ω varied around the natural frequency. The influence of the nonlinear electric field can be observed by a softening effect on the nonlinear resonance curves. Responses of the beam and the hub are presented in Fig. 3(a) and (b), respectively. We may conclude that observed nonlinearities may play essential role in near resonance operating conditions and have to be taken into account for large amplitude vibration regimes and high magnitudes of an electric field. Presented results are obtained for the symmetric structure when both beams have the same electromechanical properties. It is expected the differences in the natural modes and the resonance curves may occur when beams are slightly detuned, for example due to different length, thickness, density etc. This problem will be further investigated in future research.

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