Vibrational analogue of coherent quantum Rabi oscillations in a three-body nonlinear mechanical system

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Summary. The quantum-classical analogies are a subject of great theoretical interest because they provide the bridges between the problems similar by their physical sense but having different mathematical description. We discuss the analogy between the coherent (harmonic) quantum Rabi oscillations in a system of three coupled resonance states and nonlinear energy transfer in a classical system of three coupled nonlinear oscillators (pendulums). We show that almost complete energy exchange can occur in slow time scale between the weakly coupled subsystems in a certain range of the initial conditions. These oscillations in the energy distribution and energy exchange between the subsystems are very similar to the coherent quantum Rabi oscillations in the populations of three coupled resonance states, which were observed in a three-state Bose-Einstein condensed atomic system.

Realization of Bose-Einstein condensation in atomic gases provides samples of atoms with a macroscopic population in the ground state of the system. This population forms a coherent matter wave and is described by a macroscopic wave function, which is the solution of the nonlinear Gross-Pitaevskii equation. The analogies between coherent matter waves and coherent photons have been discussed in connection with possible realization of an "atom laser" [1]. Theoretical discussions of the atom laser have considered the case in which atoms are fed into and coupled out of the "lasing mode" continuously. The output coupler for Bose condensed atoms in a magnetic trap has been demonstrated, in which Bose condensates in a superposition of trapped and untrapped hyperfine states were created with short pulses of radio-frequency radiation, and the fraction of out-coupled atoms was adjusted between 0% and 100% by varying the amplitude of the radiation [2].

Here we discuss the vibrational analogy between the coherent quantum Rabi oscillations and the nonlinear energy transfer in a system of three coupled nonlinear oscillators (pendulums) with constant or variable parameters. In the latter case, one pair in the considered pendulum system is weakly linked with a spring while another pair is tightly bound through a common joint. We show that in both cases of constant and variable parameters the intensive energy exchange and complete energy transfer between the weakly linked subsystems can occur in a certain range of the initial conditions. These oscillations of the energy distribution and complete energy transfer between the subsystems are similar, and their time evolution is described by similar functions of time, to the quantum Rabi oscillations in the populations of three coupled resonance states. Such population oscillations were observed in a three-state system with the induced by radio-frequency radiation transitions within the ground state manifold of sodium atoms [2]. Similar periodic almost total energy exchange between three coupled nonlinear mechanical subsystems can occur only in the system with soft nonlinearity [3] but not in the system with hard nonlinearity [4]. Both analytical and numerical results of almost complete energy exchange in a three-body nonlinear mechanical system will be presented. Figures 1 and 2 show the temporal dependence of state populations P_i in the course of quantum Rabi oscillations in a system of three coupled resonance states and of the displacements and energies in a system of three coupled pendulums, respectively. In the latter case, the transition from almost complete energy exchange between the first and third pendulums via the second pendulum to energy localization in the initially excited first pendulum is also demonstrated.

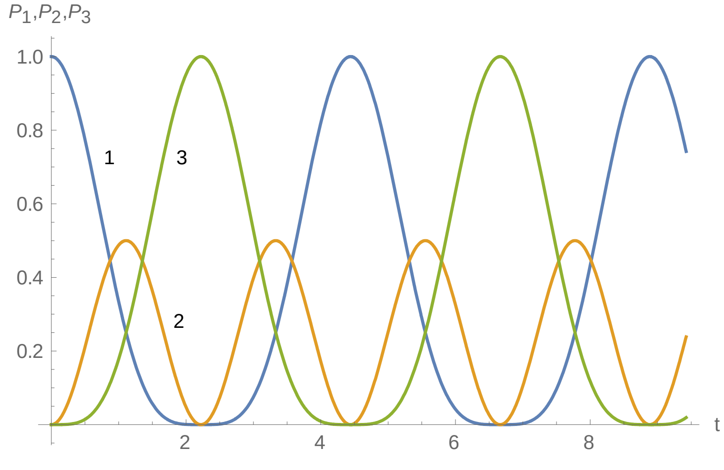

Fig. 1. Temporal dynamics of state populations P_i in the course of quantum Rabi oscillations in a system of three coupled resonance states, with the initial conditions P_1=1, P_2=0, P_3=0.

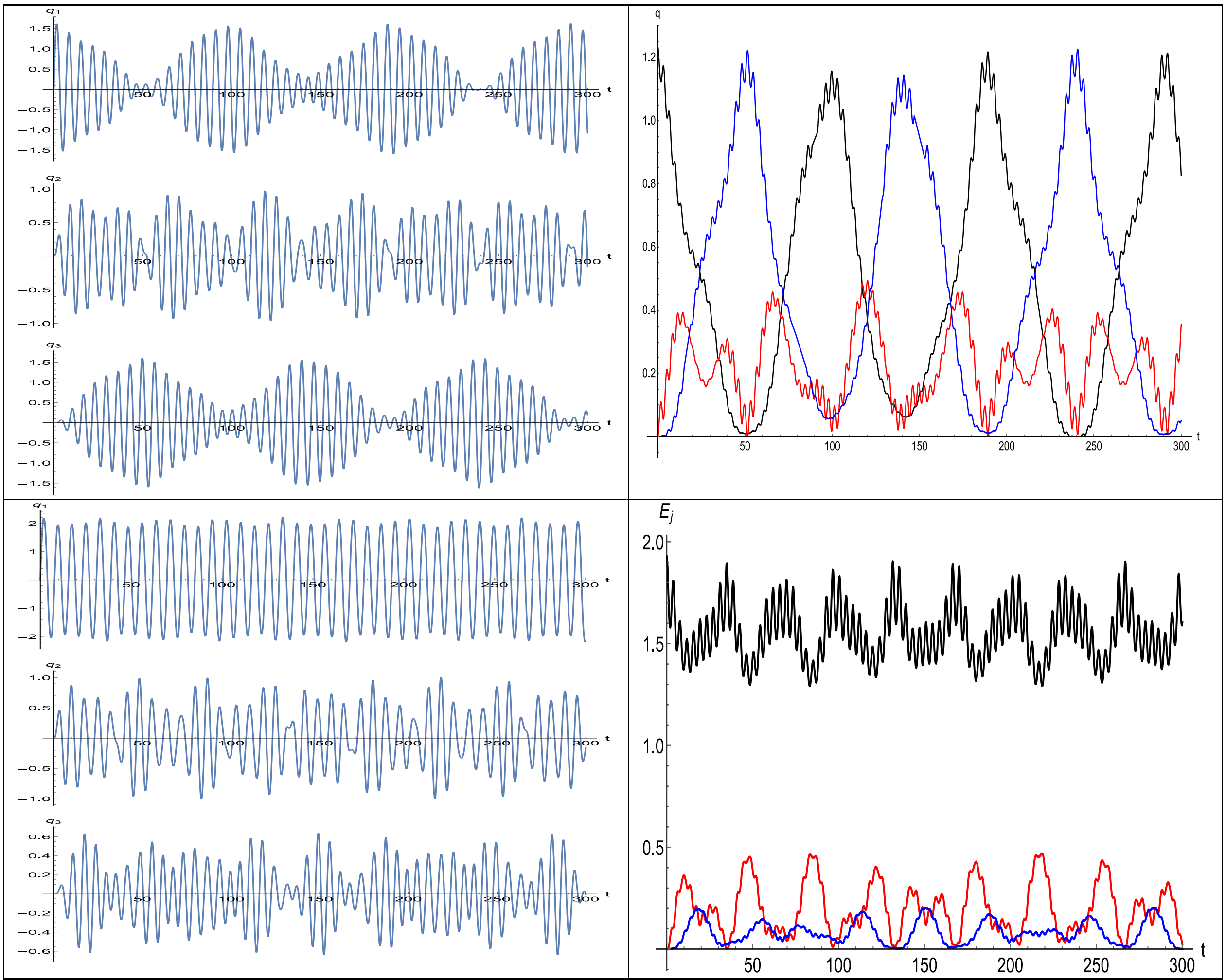

Fig.2. Temporal dependence of displacements and energies of the first, second and third coupled pendulums, which shows the transition from almost complete energy exchange between the first and third pendulums via the second pendulum (top panels) to energy localization in the initially excited first pendulum (bottom panels).

Conclusions

The described effects can be used for the designing of the devices, which can serve as controllable sources of coherent pulses of mechanical energy on the micro- and nano-scales.

References

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