A robust–tube MPC approach for the analysis of load response of power plants

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Summary. This paper addresses the inverse–dynamics/setpoint–tracking problem related to the load response of combustion-thermal power plants. Given the (discrete time) mathematical model of the plant, an approach is proposed which (if admissible) achieves the tracking of generator power and steam pressure setpoints by the control of boiler load and turbine valve which are considered as process inputs. The proposed approach bounds the one–step–ahead predicted state of the nonlinear plant model by a compact polyhedral set whose vertices are defined by Linear Time Invariant (LTI) systems. Using this, a robust–tube Model Predictive Control approach is introduced to derive the solution to the tracking problem. An application of the proposed approach is presented for a Circulating Fluidized Bed (CFB) plant.

Introduction

Current trends indicate that wind and solar energy will have a significant share in the energy production portfolio in the future. A comprehensive summary of trends and targets is provided in [4]. The widespread of renewable power of this kind reconfigures the role of conventional power production as it simultaneously triggers tightening emission and safety regulations and demanding performance requirements for combustion–thermal power plants [7, 5, 1]. Among others, these requirements involve: a) capability to change load level frequently in a fast manner within as wide range of load levels as possible, b) increased operational range by decreasing the minimum achievable load level without boiler or turbine trip, (i.e. operating for long times at partial load levels, which makes for a suboptimal operation) and c) satisfying emissions limitations both at steady-state operation and during the transient response.

To be able to satisfy these, it is essential to improve the dynamic response of power plants requiring flexible operation from (a) the firing system, (b) the water–steam system, (c) the turbine system as well as (d) the emission removal system. On the other hand, improving the dynamic response of power plants is a challenging task originating from the fact that plants are originally designed for constant load level operation according to the governing design paradigms and practice [8, 9]. The most cost–efficient way to improve the operation flexibility is to *redesign the unit master control*. Existing plants may inherently have great potential for flexibility which is currently not utilized by the controller. For example, large stored energy of a drum type boiler or the storage capacity of the superheater system can be further exploited. The balance of plant system may actively participate in controlling the generated power either applying condensate throttling technique or acting as an energy buffer.

Although the design of the process (thermal inertia, storage capacities, etc.) determines the load change capabilities, the calculation of the extremes for the plant of interest (maximum and minimum load rate of change) is of high importance. First, it helps to define the target for control design (maximum achievable ramp set point). Secondly, it provides a feedback for process design, thus may be utilized for the simultaneous design of process and control.

In the field of control design of linear uncertain systems recently robust tube–based model predictive approaches have been gaining much interest [11, 10, 6, 3]. The reported benefits (*e.g.*, computational tractability, robustness, relative simple implementability etc.) makes the tube-based problem formulation to be an attractive alternative for nonlinear systems arising in power plant control design and applications. This paper addresses the control problem related to the fast load change(s) of combustion–thermal power plants. Utilizing receding horizon control, a tube–based MPC approach is developed solving the following problem: given a dynamic power plant process model and a desired process output trajectory, what is the corresponding input trajectory (if any) subject to (safety, system and input) constraints.

The paper is organized as follows. Section *Control aspects of combustion–thermal power plants* gives a brief overview about the process of interest and related control applications. Using this, Section *Problem formulation* formulates the problem and control objectives. In Section *Approach* a robust tube–based MPC approach is detailed, while in Section entitled *Application* the performance of the proposed approach is demonstrated using a transfer function based mathematical model of a Circulating Fluidized Bed (CFB) power plant. Finally, Section *Summary and Conclusions* provides a summary of the work and draws the conclusions.

Notation

The set of non-negative real numbers is denoted by $\mathcal{R}^+$. The set of non-negative and positive integers are denoted by $\mathbb{Z}^0$ and $\mathbb{Z}^+$. Likewise, for any $K \in \mathbb{Z}^0$, $\mathbb{Z}^0_K := \{0, 1, \ldots, K\}$ and for any $K \in \mathbb{Z}^+$, $\mathbb{Z}^+_K := \{1, \ldots, K\}$. The vector $e \in R^n$ denotes the vector of ones *i.e.* $e := (1, 1, \ldots, 1)'$ where the symbol $'$ denotes the vector transposition operator. Given a vector $x \in \mathcal{R}^n$, $|x|$ denotes its Euclidean norm. A *polyhedron* is the (convex) intersection of a finite number of open and/or closed half-spaces and a *polytope* is a closed and bounded polyhedron. The symbol $\oplus$ denotes the Minkowski set addition where given two sets $\mathcal{X}, \mathcal{Y} \subset \mathcal{R}^n$, $\mathcal{X} \oplus \mathcal{Y} := \{x + y \mid x \in \mathcal{X},\, y \in \mathcal{Y}\}$. The symbol $\ominus$ denotes the Pontryagin set difference where $\mathcal{X} \ominus \mathcal{Y} := \{z \mid z \oplus \mathcal{Y} \subseteq \mathcal{X}\}$.

Control aspects of combustion–thermal power plants

Thermal power plants implement the Rankine cycle which is utilized to produce electric power from heat. Considering water as work fluid, the cycle begins with the water pump which maintains the required pressure and flow conditions in the system. After pressurization, the water is directed towards the steam generation unit where it interacts with external heat and high pressure high temperature steam is produced. The steam passes through the turbine which drives the electric generator. After the turbine, the steam is condensed and the water leaving the condenser enters the pumping unit closing the cycle. The heat required to empower the cycle is produced in the furnace by combusting a given fuel. Figure 1 depicts the main building blocks of the process.

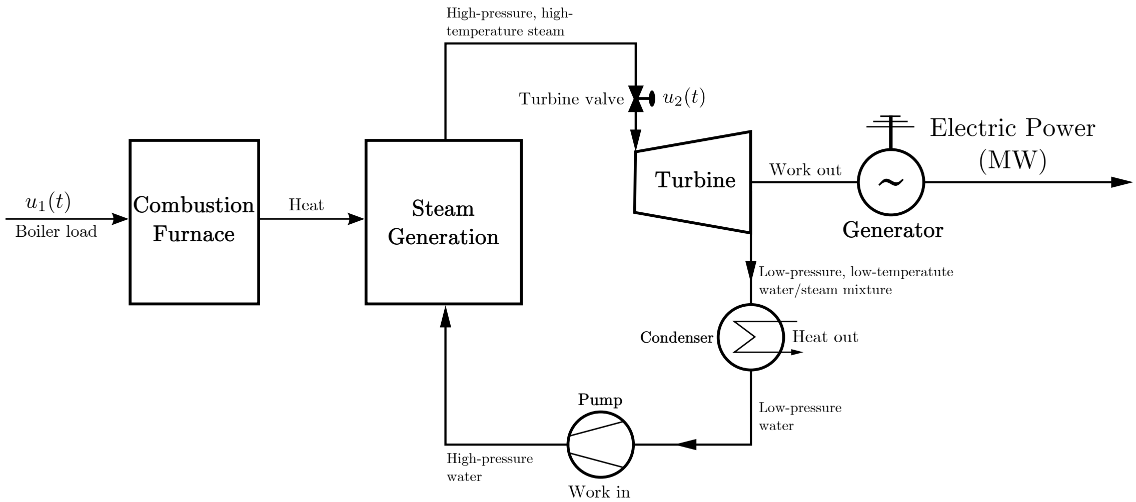

Figure 1: Process of interest (schematic).

The main objective of a power plant is to provide regulating power according to the actual balance between power supply and demand. From the perspective of control, the regulating power appears as a time varying setpoint for the generator power and steam pressure of the plant which are considered as (main) process outputs. Correspondingly, the the main process inputs are the load of the boiler which is proportional to the heat release and the opening degree of the valve located at the inlet of the turbine (referred to as turbine valve).

In practice, the provision of regulating power involves rapid load changes and it is achieved by the control of the two main process inputs. Regarding the control, industrial solutions favor one of the following strategies: (1) Boiler follow, (2) Turbine follow and (3) Coordinated control. The key idea for the first two (1 and 2) is to couple the setpoints with process inputs and achieve the tracking of a particular setpoint with the manipulation of the corresponding input. With that said, in boiler follow mode, the furnace is disconnected from the power generation control, its load level is adjusted to ensure the tracking of the pressure setpoint while with the manipulation of the turbine valve the steam turbine is able to utilize the stored energy in the system to provide immediate response to the power demand. In contrast, the turbine follow control strategy interchanges the coupling as the furnace fires to satisfy the power demand and turbine valve maintains the pressure. Unlike the former control strategies, coordinated control do not use coupling it aims to manipulate the system inputs in synergy. In general, it includes various logic schemes to open/close the steam turbine valves for quick load response as well as fire the furnace for the anticipated energy requirements while tracking the steam pressure setpoint.

The demanding performance requirements magnify the need for coordinated control for combustion–thermal power plants. In practice, the design of coordinated control requires expertise as the control law is usually derived by means of mass/energy balances which highly rely on process specific knowledge.

One of the main objectives of this paper is to approach the design of coordinated control in purely mathematical manner assuming that the mathematical model of the process which maps the boiler load $u_1(t)$ and the opening degree of the turbine valve $u_2(t)$ to the generator power $y_1(t)$ and operating pressure $y_2(t)$ is known but the process knowledge (physical interpretation of the mathematical model) is not available. Results of this approach are of high interest for comparison with the process knowledge based methods.

Problem formulation

Process model

The dynamic behavior of the system of interest can be approximated by lumped models *i.e.* ordinary differential equation(s). However, since the control is usually piecewise constant (i.e., digital control is applied) the system is modeled in discrete time by the recursive formula

$$x_{k+1} = f(x_k, u_k), \quad k = 0, 1, \ldots \tag{1a}$$
$$y_k = Cx_k + y_{\mathrm{c}} \tag{1b}$$

where $x_k \in \mathcal{R}^n$ is the system's state, $u_k \in \mathcal{R}^2$ is the vector of manipulated inputs and $y_k \in \mathcal{R}^2$ is the system output vector at time $k\Delta t$ where $\Delta t > 0$ is the time discretization unit. The vector y_{c} represents a constant threshold. The system variables x and u are subject to hard (state and input) constraints

$$x_k \in \mathcal{X}_k \quad \text{and} \quad u_k \in \mathcal{U}, \quad \forall k \in \mathbb{Z}^0 \tag{2}$$

which are characterized by nonempty polytopes $\mathcal{X}_k = \{x \in \mathcal{R}^n \mid A_k x \leq b_k\}$ and $\mathcal{U} = \{u \in \mathcal{R}^2 \mid A_{\mathrm{u}} u \leq b_{\mathrm{u}}\}$. The defined sets $\mathcal{X}_k$ and $\mathcal{U}$ represent the operational constraints of the power plant of interest.

Modeling remarks

Power plants are complex, distributed physical systems. Relying on (modeling) assumptions which provide an approximation of the underlying physical phenomena, the lumped model of interest (ODE representation) is usually derived (a) by discretizing the elements of the plant in the spatial dimension and applying conservation laws (mass, momentum and energy) for the discrete elements or (b) simple transfer function description of the process components are used to form the complete model of the plant. The resulted model accounts for the significant transport delay with respect to the process input(s). Regarding this, the state variable of model (1) is a compound as the transport delay is augmented to the state applying zero order hold (ZOH) on the inputs subject to time discretization. Correspondingly, unlike the dimension of the system inputs and outputs the dimension of the state n is not specified in the problem setting phase, it depends on the chosen modeling approach and its settings.

Inverse–dynamics/setpoint–tracking problem

This paper addresses the problem of load response of power plants. From engineering perspective this can be associated with the problem of (1) qualifying how well power plant process outputs follow pre-defined time varying signals (setpoints) and (2) how to make the process outputs follow pre-defined time varying signals. To capture the essence of load response problem we consider *inverse–dynamics* and *setpoint–tracking* problem classes.

Definition 1 (Inverse–dynamics problem)**.** *Given a finite lookahead horizon* $K \in \mathbb{Z}^+$ *and a reference trajectory* $\{r_1, r_2, \ldots, r_K\}$ *find an input trajectory* $\{u_0, u_1, \ldots, u_{K-1}\}$ *such that* $\max\{|y_1 - r_1|, |y_2 - r_2|, \ldots, |y_K - r_K|\} = 0$ *subject to* (1) *and* (2)*.*

Correspondingly,

Definition 2 (Setpoint–tracking problem)**.** *Given a finite lookahead horizon* $K \in \mathbb{Z}^+$ *and a reference trajectory* $\{r_1, r_2, \ldots, r_K\}$ *find an input trajectory* $\{u_0, u_1, \ldots, u_{K-1}\}$ *so that* $\exists k \in \mathbb{Z}^0_K$ *and finite* $\delta, \varepsilon \in \mathcal{R}^+$ *for which* $\max\{|y_0 - r_0|, |y_1 - r_1|, \ldots, |y_{k-1} - r_{k-1}|\} \leq \delta$ *and* $\max\{|y_k - r_k|, |y_{k+1} - r_{k+1}|, \ldots, |y_K - r_K|\} \leq \varepsilon$ *subject to* (1) *and* (2) *where* ε *is a sufficiently small tolerance (in engineering sense).*

The relation between the two problem classes are clearly highlighted by their definitions, as setpoint–tracking can be considered as a relaxation of the inverse–dynamics problem. The main objective of this paper is to provide an approach which renders a solution to the inverse dynamics problem. However, this problem class cover only a small portion of real life situations which makes the relaxation (and the derivation of a solution approach for the relaxed problem) necessary.

Approach

Given the process model and the initial state x_0, the future evolution of the system for a given control sequence $\{u_0, u_1, \ldots, u_{K-1}\}$ can be determined simply by applying the recursive formula (1). The related inverse problem however involves numerous challenges originating form the fact that, for nonlinear high dimensional systems the corresponding mathematical problem usually results in non-convex optimization which turns out to be computationally challenging/intractable. To resolve this issue, in this section a solution approach for the inverse problem is proposed relying on the concepts and methods of the field of robust Model Predictive Control (MPC). The approach relies on perfect state information, thus we begin with the formulation of the following assumption:

Assumption 1. *The state is measured, thus* x_k *is known at every time instant* $k \in \mathbb{Z}^0$

Let $\hat{\mathcal{X}}_k$ denote the set of states which must be reached at time k where

$$\hat{\mathcal{X}}_k := \{x \in \mathcal{X}_k \mid Cx = r_k - y_{\mathrm{c}}\}, \quad \forall k \in \mathbb{Z}^0_K \tag{3}$$

and r_k is the reference signal (setpoint) which must be tracked by the output y_k. Formula (3) is the mathematical equivalent to the constraint $y_k = r_k$ where $\hat{\mathcal{X}}_k$ is going to be referred to as *target set* thoughout this paper. Since $\mathcal{X}_k$ are polytopic definition (3) implies that $\hat{\mathcal{X}}_k$ are polytopic sets as well for all $k \in \mathbb{Z}^0_K$.

The introduction of the target sets makes possible to assign a geometrical meaning to the inverse/tracking problem. As the target sets are compact, the solution require the state trajectory $\{x_0, \dots, x_K\}$ to be within a "tube" referred to as *target tube* defined by a sequence of sets of states $\{\hat{\mathcal{X}}_0, \dots, \hat{\mathcal{X}}_K\}$. Using this, the equivalent mathematical problem is a target tube reachability/constraint satisfaction problem of the form: given $x_0 \in \hat{\mathcal{X}}_0$ and a finite lookahead horizon $K \in \mathbb{Z}^+$,

$$\text{find } \Big\{u_k \mid u_k \in \mathcal{U},\ x_{k+1} = f(x_k, u_k),\ x_{k+1} \in \hat{\mathcal{X}}_{k+1},\ \forall k \in \mathbb{Z}^0_{K-1}\Big\} \tag{4}$$

In the sequel a solution to (4) is considered as the solution to the inverse/tracking problem. As already pointed out, the derivation of the solution to a problem of this form might be prohibitively difficult (especially when no knowledge about the underlying physical phenomena is utilized). To make the corresponding mathematical problem computationally tractable the following assumption is introduced:

Assumption 2. *There exists a finite $N \in \mathbb{Z}^+ \setminus \{1\}$ and controllable pairs $[A_j\ B_j]$, $A_j \in \mathcal{R}^{n\times n}$ and $B_j \in \mathcal{R}^{n\times 2}$ such that $f(x_k, u_k) \in \Omega(x_k, u_k)$ for all $(x_k, u_k, k) \in (\mathcal{X}_k \times \mathcal{U} \times \mathbb{Z}^0)$ where $\Omega(x_k, u_k) := \mathrm{Conv}\{A_j x_k + B_j u_k,\ j \in \mathbb{Z}^+_N\}$, $\mathrm{Conv}\{\dots\}$ denotes convex hull and $A_j x_k + B_j u_k$ are the vertices of the convex hull.*

Assumption 2 implies that $f(x_k, u_k)$ can be expressed as the convex combination of the vertices of $\Omega(x_k, u_k)$, that is, there exist multipliers $\lambda_j(x_k, u_k) \in \mathcal{R}$ such that

$$f(x_k, u_k) = \sum_{j=1}^{N} (A_j x_k + B_j u_k)\lambda_j, \quad \sum_{j=1}^{N} \lambda_j = 1 \quad \text{and} \quad 0 \le \lambda_j \le 1, \quad \forall j \in \mathbb{Z}^+_N. \tag{5}$$

The key idea of this formulation is to bound the one step ahead prediction of the nonlinear mathematical model (spread of the state trajectory) by a nonempty polytopic set $\Omega(x_k, u_k)$ whose vertices are formed by Linear Time Invariant (LTI) systems. The advantage given by the use of LTI systems is that these are solvable analytically. This properly is exploited by the approach which utilizes the predicted trajectory of the bounding LTI systems to solve (4).
Mathematically, one of the main concerns with the target tube reachability problem (4) is that, the existence of target tube do not provide any information about the existence of solution(s). However, if the target tube exists problem solvability can be approached by the calculation of the robust control invariant tube which usually poses a stricter constraint system by introducing a "tube within the target tube" [2]. The robust control invariant tube is calculated using the bounding LTI systems.

Definition 3. *A sequence of sets of states (tube) $\{\bar{\mathcal{X}}_0, \dots, \bar{\mathcal{X}}_K\}$ is robust control invariant for the system $x_{k+1} = f(x_k, u_k)$ and constraints $\hat{\mathcal{X}}_k$, $\mathcal{U}$ if and only if $\bar{\mathcal{X}}_k \subseteq \hat{\mathcal{X}}_k$ and $\forall x_k \in \bar{\mathcal{X}}_k\ \exists u_k \in \mathcal{U}$ such that $(A_j x_k + B_j u_k) \in \bar{\mathcal{X}}_{k+1}\ \forall j \in \mathbb{Z}^+_N$.*

By definition, the roust control invariant tube for $k = K$

$$\bar{\mathcal{X}}_K = \hat{\mathcal{X}}_K \tag{6}$$

and for $k = K-1, \dots, 0$

$$\bar{\mathcal{X}}_k = \Big\{x \in \hat{\mathcal{X}}_k \mid \exists u \in \mathcal{U} \text{ such that } (A_j x + B_j u) \in \bar{\mathcal{X}}_{k+1}\ \forall j \in \mathbb{Z}^+_N\Big\} \tag{7}$$

As indicated, the calculation of the robust control invariant tube proceeds backwards and performed off-line. Since $\hat{\mathcal{X}}_k$ are polytopic sets, definition (7) implies that $\bar{\mathcal{X}}_k$ are polytopic as well for all $k \in \mathbb{Z}^0_K$. Consequently, the control invariant tube can be derived using linear programming approaches.
Once obtained, the control invariant tube provides some information about the existence of solutions to the inverse problem. Formally,

Proposition 1 (Sufficiency). *If $\bar{\mathcal{X}}_k \ne \emptyset\ \forall k \in \mathbb{Z}^0_K$ and $x_0 \in \bar{\mathcal{X}}_0$ then* (4) *has a solution.*

Proof. If $x_k \in \bar{\mathcal{X}}_k$ definition (7) implies that $\exists u_k \in \mathcal{U}$ such that $\Omega(x_k, u_k) \in \bar{\mathcal{X}}_{k+1}$ where $\Omega(x_k, u_k) = \mathrm{Conv}\{A_j x_k + B_j u_k,\ j \in \mathbb{Z}^+_N\}$ thus $f(x_k, u_k) \in \bar{\mathcal{X}}_{k+1}$ since $f(x_k, u_k) \in \Omega(x_k, u_k)$, for all $k \in \mathbb{Z}^0_{K-1}$. By the principle of induction if $x_0 \in \bar{\mathcal{X}}_0$ there exists an input sequence $\{u_0, \dots, u_{K-1}\}$ such that $\{x_1, \dots, x_K\} \in \{\bar{\mathcal{X}}_1, \dots, \bar{\mathcal{X}}_K\} \subseteq \{\hat{\mathcal{X}}_1, \dots, \hat{\mathcal{X}}_K\}$ subject to (1a) which completes the proof. □

Although it gives a sufficient condition for solvability, the calculation of control invariant tube is not constructive since it do not provide the solution (input sequence) to the formulated problem. However, in case the control invariant tube exists the corresponding constraint satisfaction problem (4) can be efficiently solved by linear/quadratic programming approaches. To formulate the solution strategies we rely on the following assumption.

Assumption 3. $\bar{\mathcal{X}}_k \ne \emptyset$, $\forall k \in \mathbb{Z}^0_K$ *and* $x_0 \in \bar{\mathcal{X}}_0$.

Using this, let us consider the following MPC strategy (MPC1): at each sampling instant $k = 0, 1, \dots, K-1$

1. Measure the value of the state vector x_k.
2. Determine the input $u_k^\star = \arg\min \{e' u_k \mid A_j x_k + B_j u_k \in \bar{\mathcal{X}}_{k+1},\, u_k \in \mathcal{U} \quad \forall j \in \mathbb{Z}_N^+\}$.
3. Apply the control input $u_k = u_k^\star$ to (1).

Due to the fact that $\bar{\mathcal{X}}_k$ and $\mathcal{U}$ are polytopic sets the optimization problem defined in step 2 is a linear program. If assumption 3 holds it follows from proposition 1 that the linear program has a feasible solution at each timestep k, thus the presented MPC strategy MPC1 solves (4).
The formulated linear program uses a one step prediction of the system trajectory (lookahead) which makes the solution approach computationally efficient due to the small number of decision variables and constraints. However, computational efficiency comes with the price that the produced solutions may not be accepted by design engineers since the approach based on one step lookahead might involve problems regarding the smoothness of the solutions. To overcome this difficulty, an approach is proposed in the next section which incorporates smoothness criteria into the cost function and uses multistep lookahead.

Smoothness objective
Let us define the ”nominal system”

$$\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}u_k \tag{8}$$

where the system matrices $[\bar{A}\ \bar{B}] = N^{-1}\sum_{j=1}^{N}[A_j\ B_j]$ are the arithmetic mean of the vertices $[A_j\ B_j]$ and $\bar{x}_k \in \mathcal{R}^n$ is the state of the nominal system. Given an initial state x_k and input u_k the successor state given by the nominal dynamics is located at the ”center” of $\Omega(x_k, u_k)$ which bounds the one step lookahead spread of the successor state given by the dynamics (1a). From this perspective, (1a) can be viewed as an ”uncertain system” with the dynamics

$$x_{k+1} = \bar{A}x_k + \bar{B}u_k + w_k \tag{9}$$

where $w_k \in (\Omega(x_k, u_k) \ominus (\bar{A}\bar{x}_k + \bar{B}u_k))$ (regarded as disturbance) captures the mismatch between (1a) and (8). Using this, let us consider following program

$$\begin{aligned}
&\underset{\{u_0,\dots,u_{L-1}\}}{\text{minimize}} && \sum_{l=0}^{L-1} |u_{l+1} - u_l|^2 && && \text{(10a)}\\
&\text{subject to} && \bar{x}_0 - x_k && = 0 && \text{(10b)}\\
& && \bar{x}_{l+1} - \bar{A}\bar{x}_l - \bar{B}u_l && = 0 && \text{(10c)}\\
& && A_j\bar{x}_l + B_j u_l \in \bar{\mathcal{X}}_{k+l+1} && \forall j \in \mathbb{Z}_N^+ && \text{(10d)}\\
& && u_l \in \mathcal{U} && \forall l \in \mathbb{Z}_{L-1}^0 && \text{(10e)}
\end{aligned}$$

and the MPC strategy (MPC2): at each sampling instant $k = 0, 1, \dots, K-1$

1. Measure the value of the state vector x_k.
2. Specify the lookahead horizon L.
3. Determine the input sequence $\{u_0^\star, \dots, u_{L-1}^\star\}$ which solves (10).
4. Apply the control input $u_k = u_0^\star$ to (1).

Due to the fact that $\bar{\mathcal{X}}_k$ and $\mathcal{U}$ are polytopic sets (10) is a quadratic program. The objective function determines a smoothness condition for the input sequence $\{u_0, \dots, u_{L-1}\}$ on a given lookahead horizon L. If assumption 3 holds, it follows from proposition 1 that (10) has a feasible solution at each timestep k, thus the presented MPC strategy MPC2 solves (4) (see proof of proposition 1).

Relaxed/setpoint–tracking problem
The derivation of the solution approach is based on assumption 3 demanding the existence of the control invariant tube. However, it turns out that most of the cases (especially in a request for rapid load changes) the control invariant tube for system (1) does not exist if the target sets are defined by 3 which implements strict equality. Consequently, on one hand, the solution to the inverse dynamics problem can not be rendered by the proposed MPC strategies (MPC1 and MPC2) while on the other in such cases the problem may not have a solution at all which is well known by design engineers revealed by studies using process specific knowledge.
In such cases it is desirable to satisfy one of the tracking objectives (generator power or steam pressure). A common engineering approach to choose this objective is based on preference ordering where the objectives are ranked according

to their (relative) importance. The satisfaction of the objective having higher importance is the main goal while the other objective can deviate but should remain as "close" to the desired one as possible. The problem of interest has two objectives which are ordered as follows: tracking of (1) generator power setpoint and (2) pressure setpoint. From power plant operational point of view this means that during transients the satisfaction of the demand (the production of the required electric power) is the main priority and the steam pressure is allowed to fluctuate within acceptable bounds. In steady state operation however no deviation from the setpoints are allowed.
Utilizing preference ordering this section proposes an approach which relaxes (3) and formulates an optimization problem for setpoint tracking. By expressing (1b) the (scalar) objectives in a preference order are

$$\begin{bmatrix} r_{k,1} \\ r_{k,2} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x_k + \begin{bmatrix} y_{\mathrm{c},1} \\ y_{\mathrm{c},2} \end{bmatrix}. \tag{11}$$

Using this, the target set for the relaxed problem is defined as follows:

$$\hat{\mathcal{X}}_k := \{x \in \mathcal{X}_k \mid |C_1 x + y_{\mathrm{c},1} - r_{k,1}| \leq \varepsilon,\ \underline{r}_{k,2} \leq C_2 x + y_{\mathrm{c},2} \leq \bar{r}_{k,2}\}, \quad \forall k \in \mathbb{Z}_K^0 \tag{12}$$

where the tolerance $\varepsilon > 0$ is a sufficiently small parameter and $\underline{r}_{k,2}$, $\bar{r}_{k,2}$ are the upper and lower boundaries for the second objective. While the parameter ε is problem dependent representing meaningful or acceptable bound for constraint satisfaction from engineering perspective, the pressure boundaries $\underline{r}_{k,2}$ and $\bar{r}_{k,2}$ are well defined and given by the system operators. Since both objectives are scalar, $\hat{\mathcal{X}}_k$ are polytopic sets $\forall k \in \mathbb{Z}_K^0$.
Given ε, $\underline{r}_{k,2}$ and $\bar{r}_{k,2}$, $\forall k \in \mathbb{Z}_K^0$ similarly to the former approaches the robust control invariant tube is calculated according to (6) and (7) using definition (12) to define the target sets. In the sequel, we restrict our attention to the case when the robust control invariant tube for the relaxed problem exists (i.e. target sets are defined according to (12) and assumption 3 holds) where proposition 1 guarantees the solvability. On the other hand, it must be pointed out that, since proposition 1 gives a sufficient condition the non–existence of the robust control invariant tube do not imply that the problem do not have a solution. However, the investigation os such case (robust control invariant tube does not exist) is out of scope of the current paper. Similarly to (10) the tracking of the pressure setpoint (2nd objective) is formulated as an objective function of the following program

$$\begin{aligned}
&\underset{\{u_0,\dots,u_{L-1}\}}{\text{minimize}} && \sum_{j=1}^{N}\sum_{l=0}^{L-1} (y_l - r_{k+l,2})^2 && && \text{(13a)}\\
&\text{subject to} && \bar{x}_0 - x_k && = 0 && \text{(13b)}\\
& && \bar{x}_{l+1} - \bar{A}\bar{x}_l - \bar{B}u_l && = 0 && \text{(13c)}\\
& && y_l - C_2 x_l - y_{\mathrm{c},2} && = 0 && \text{(13d)}\\
& && A_j \bar{x}_l + B_j u_l \in \bar{\mathcal{X}}_{k+l+1} && \forall j \in \mathbb{Z}_N^+ && \text{(13e)}\\
& && u_l \in \mathcal{U} && \forall l \in \mathbb{Z}_{L-1}^0 && \text{(13f)}
\end{aligned}$$

Since $\bar{\mathcal{X}}_k$ and $\mathcal{U}$ are polytopic $\forall k \in \mathbb{Z}_K^0$ (13) is a quadratic program. Using this, let us consider the following MPC strategy (MPCR): at each sampling instant $k = 0, 1, \dots, K-1$

1. Obtain the value of the state vector x_k.
2. Specify the lookahead horizon L.
3. Determine the input sequence $\{u_0^\star, \dots, u_{L-1}^\star\}$ which solves (13).
4. Apply the control input $u_k = u_0^\star$ to (1).

Proposition 1 implies that (13) has a feasible solution at each timestep k, thus the presented MPC strategy MPCR solves the relaxed problem (see proof of proposition 1).

Application

The proposed approach was tested with a mathematical model describing the dynamic behavior of a Circulating Fluidized Bed (CFB) power plant. The plant model built up by transfer function blocks under MATLAB Simulink capturing the following relationships: fuel power – steam generation (thermal inertia), steam storage – steam pressure (storage capacity) and the dynamics of turbo–generator unit. Using $\Delta t = 2$ (sec) time discretization the sate space description resulted in the following bilinear model

$$\begin{aligned}
x_{k+1} &= A_0 x_k + u_{2,k} A_1 x_k + B u_k && \text{(14a)}\\
y_k &= C x_k + y_{\mathrm{c}} && \text{(14b)}
\end{aligned}$$

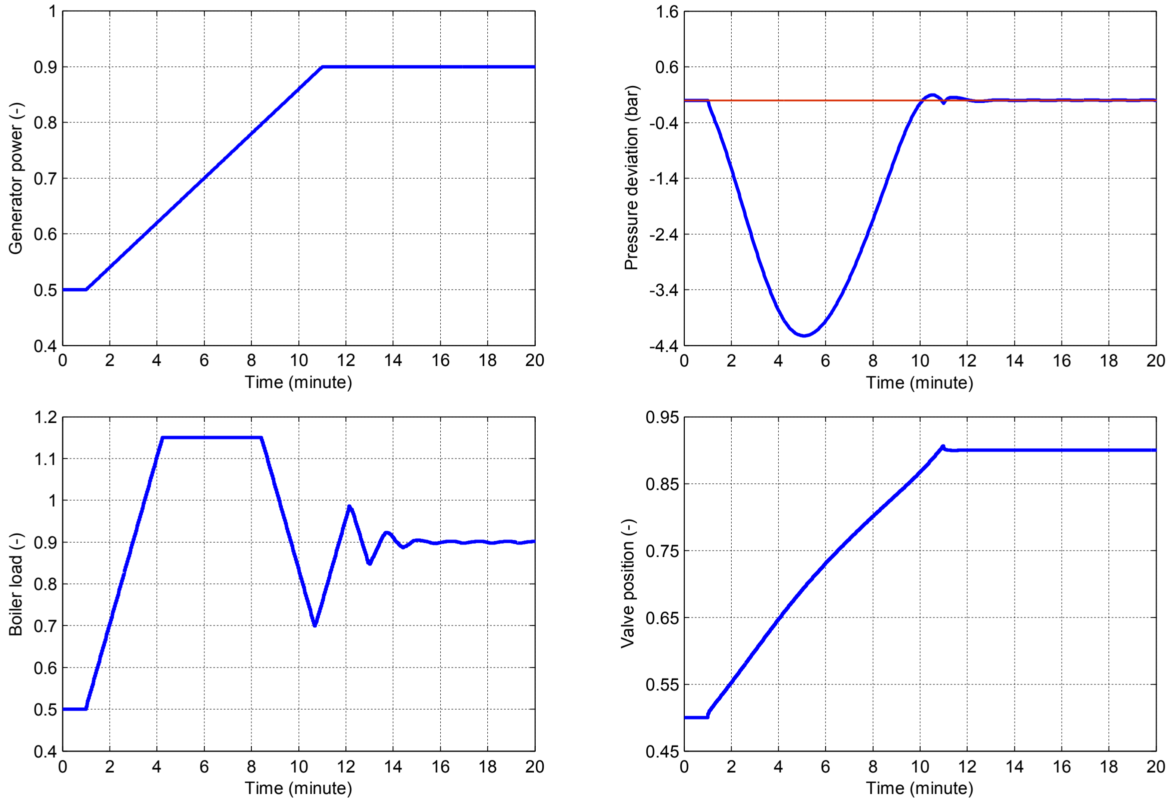

Figure 2: Results of the tracking problem problem generated by the MPCR approach for a CFB plant at 4% MCR/min load change. The allowed pressure deviation was ± 5 (Bar). Process output charts are located at the top while process input charts are at the bottom of the figure. Setpoints are highlighted by red color.

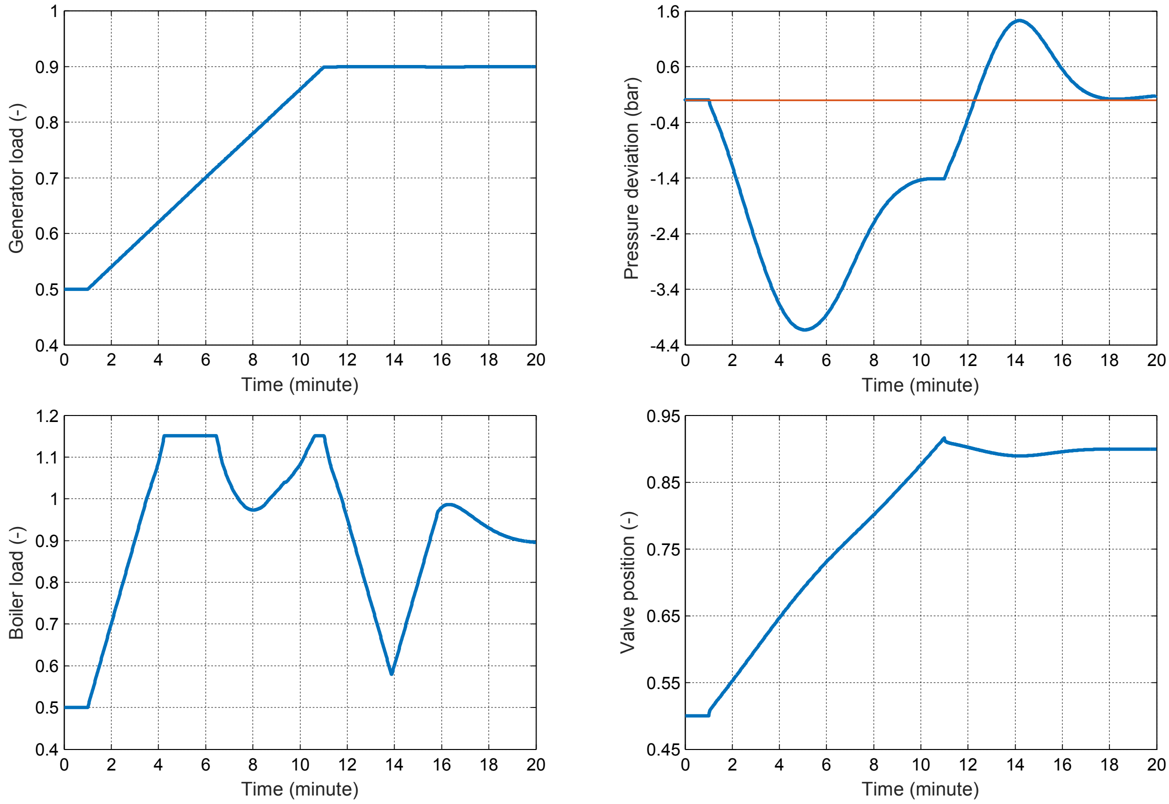

Figure 3: Results of the tracking problem problem generated by the PID control approach for a CFB plant at 4% MCR/min load change. The allowed pressure deviation was ± 5 (Bar). Process output charts are located at the top while process input charts are at the bottom of the figure. Setpoints are highlighted by red color.

including an 11 dimensional state vector ($x \in \mathcal{R}^{11}$). For this particular case, the restriction on the valve opening $u_{2,k} \in [0, 1]$, $\forall k \in \mathbb{Z}^0$ makes the derivation of the bounding LTI systems straightforward. It is easy to see that (14a) is bounded by the two LTI systems: $A_0 x_k + B u_k$ and $(A_0 + A_1) x_k + B u_k$.

The possible load demand scenario for this plant vary in a wide range. Usual load ramps are in the range of 2-4% MCR/min (percentage of the maximum continuous rate per time) however, the requirements are continuously tightening. Using this, the tested scenario was a 40% load change from 50% to 90% MCR with a ramp of 4% MCR/min on a 20 minute time window. This scenario required the application of MPCR strategy which solves the relaxed problem. The MPCR run with the following parameter settings: prediction horizon 10 min ($L = 300$ steps) and constraint satisfaction tolerance $\varepsilon = 0.05\%$. Figure 2 plots the input–output trajectories generated by the MPCR strategy for the outlined scenario.

Given the process model, a common approach by design engineers to solve the related problem in a ”mathematical sense” is the application of PID controller(s). As the problem is constrained some challenges are introduced for the PID tuning. Figure 3 plots the input–output trajectories generated by a ”typical” PID controller which is used by design engineers for design, development and research purposes.

As can be seen both approaches satisfy the requirements for generator power setpoint tracking however the tracking of the pressure signal and the smoothness of the boiler load input trajectory is less impressive if PID control is used. However, it must be pointed out that the above comparison has only informative significance since no parameter tuning was performed to find the ”optimal” values of the PID controller for the test scenario. On the other hand, the main purpose of the presented examples is to highlight the potential of the proposed approach while comparison with other approaches and benchmarking is out of the scope of the current paper.

Summary and Conclusions

This paper proposes a tube–based robust model predictive control approach to solve the inverse–dynamics/setpoint–tracking problems related to the operation of power plants. The problem reads as follows: given a power plant process model, how to manipulate the process inputs (boiler load and turbine valve opening) so that the process outputs (generator power and steam pressure) follow a pre–determined (time varying) setpoint. One of the main objectives of this paper was to provide a solution approach to the outlined problem in a ”purely mathematical” manner, that is, the mathematical model of the process is known but the related process knowledge (physical interpretation of the mathematical model) is not available.

Given the discrete time mathematical model model, the key idea of the proposed approach is to bound the spread of the one step ahead predicted state of the nonlinear plant by a polytope whose vertices are set by linear time invariant systems. Relying on this idea, robust control invariant tubes for the nonlinear system are defined and sufficient condition for the solvability of the problem is formulated. Using this, three different MPC strategies were proposed, two for the inverse problem (with and without input trajectory smoothness condition) and one for the tracking which was formulated by relaxation (assuming that the inverse problem has no solution). A setpoint tracking application was presented using a transfer function model of a Circulating Fluidized Bed (CFB) power plant.

The proposed approach was formulated without the loss of generality thus it is applicable for plant models of any detail (including finite volume models as well as transfer function description). Under mild assumptions the approach provide a computationally tractable alternative to handle power plant inverse–dynamics and setpoint–tracking problems.

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References

[1] Alobaid F., Mertens N., Starkloff R., Lanz T., Heinze C., Epple B. (2016) Progress in dynamic simulation of thermal power plants. *International Journal of Progress in Energy and Combustion Science*, **59**:79–162.

[2] Bertsekas D.P. (2005) *Dynamic Programming and Optimal Control*, Vol 1. Athena Scientific.

[3] Bumroongsri P., Kheawhom S. (2012) An off–line robust MPC algorithm for uncertain polytopic discrete-time systems using polyhedral invariant sets. *Journal of Process Control*, **22**:975–983.

[4] Child M., Breyer C.H. (2016) Vision and initial feasibility analysis of a recarbonised Finnish energy system for 2050. *Renewable and Sustainable Energy Reviews*, **66**:517–536.

[5] Eurelectric Group. (2015) Thermal power plants – a vital asset in a new energy world. *Technical report*, Union of the Electricity Industry.

[6] Gonzalez R., Fiacchini M., Alamo T., Guzman J.L., Rodriguez F. (2011) Online robust tube-based MPC for time-varying systems: a practical approach. *International Journal of Control*, **84**(6):1157–1170.

[7] Henderson C. (2014) Increasing the flexibility of coal–fired power plants. *Technical report*, IEA Clean Coal Centre.

[8] Kovács J., Kettunen A., Ikonen E., Hultgren M., Niva L. (2015) Addressing the challenge of fast load change requirements. In proceedings of the *22nd International Conference on Fluidized Bed Combustion*.

[9] Kovács J., Kettunen A., Selek I. (2017) Dynamic behaviour of a commercial CFB unit – measurements and related studies. In proceedings of the *12th International Conference on Fluidized Bed Technology*, (in print).

[10] Limon D., Alvarado I., Alamo T., Camacho E.F. (2010) Robust tube-based MPC for tracking of constrained linear systems with additive disturbances. *Journal of Process Control*, **20**:248–260.

[11] Mayne D.Q., Seronb M.M., Rakovic S.V. (2005) Robust model predictive control of constrained linear systems with bounded disturbances. *Automatica*, **41**:219–224.