Low-dimensional piecewise smooth maps with an unpredictable number of switching manifolds

Viktor Avrutin*, Zhanybai T. Zhusubaliyev** and Erik Mosekilde***
**Institute for Systems Theory and Automatic Control, University of Stuttgart, Germany*
***Department of Computer Science, Southwest State University, Kursk, Russia*
****Department of Physics, The Technical University of Denmark, Lyngby, Denmark*

Summary. It is well-known that piecewise-smooth systems are extremely challenging from a mathematical point of view, in particular because of the bifurcation phenomena caused by interactions between invariant sets and switching manifolds. As a first necessary step, the theory of piecewise-smooth systems has been developed for models with one switching manifold. However, this is not always sufficient for practical applications. In particular, a worldwide growing interest in renewable energy sources (such as solar photovoltaic and wind energy systems) as well as electric car drives led in the last years to an increasing interest in dynamics of power electronic DC/AC converters. However, modeling of DC/AC converters leads to piecewise smooth stroboscopic maps whose properties still remain to be examined in detail. A distinguishing feature of these maps is their extremely high (and practically unpredictable) number of switching manifolds. This causes several unusual bifurcations phenomena to occur. We report how the transition from the domain where the stroboscopic map has stable and globally attracting fixed points to the region of chaotic dynamics occurs via irregular cascades of border collisions. We show how some of these border collisions form complex patterns in the stable domain, and how smooth (pitchfork and flip) bifurcations of different fixed points form (under certain consitions) macroscopic patterns stratching across the overall bifurcation structure.

Power converters: from DC/DC to DC/AC and AC/DC cases

Power converters with switching operation are important practical applications of the theory of piecewise-smooth dynamical systems. Since electrical sources can be either DC (direct current) or AC (alternating current) there are four basic types of converters: DC/DC, AC/DC, DC/AC and AC/AC. Initially, the main research activities in nonlinear dynamics applied to power electronics were focused on DC/DC converters, as their mathematical models were known to be easier than for other types. However, during the last few years, the number of studies devoted to nonlinear phenomena in DC/AC and AC/DC converters was rapidly growing. Presently, there exists a large demand for theoretical results regarding the dynamic behavior of these systems. Indeed, the research in this area is strongly motivated by industrial applications, in particular, due to a worldwide growing interest on renewable energy sources, such as solar photovoltaic and wind energy systems, as well as on power supplies and drives of electric and hybrid cars. It is obviously of interest to contribute to the development of a broader theoretical foundation that can shed light on some of the unusual phenomena already observed [1, 3, 4]. Surprisingly, modeling DC/AC and AC/DC converters leads to a class of piecewise-smooth systems which have never been investigated before. In the following we focus our work on DC/AC converters (also called inverters), although the basic principles of the modeling procedures apply to AC/DC converters as well (see [2] for an example).

Modeling procedure

To explain the challenges of the modeling of DC/AC converters, let us recall some basic differences in the activation of the switching dynamics in DC/DC and DC/AC converters and the resulting differences in the properties of their models. Recall that the control of a DC/DC converter involves *one* single frequency (the switching frequency), while the reference signal is constant and only specifies the value of the desired DC output voltage. By contrast, the control of a DC/AC converter involves *two* frequencies, namely the switching frequency and the frequency of the sinusoidal reference signal that defines the frequency and phase of the desired AC output voltage. Therefore, for both types of circuit, a low-dimensional stroboscopic map f can be defined over one switching period. In the simplest case, this map has two switching manifolds related to saturation at the beginning and end of the switching period. Depending on the applied pulse-width modulated control and the circuit topology, a few more switching manifolds may be present in the model.

For DC/DC converters, the map f already represents the final model. For DC/AC converters, on the other hand, the low frequency of the reference signal (typically 50 Hz) must be reproduced in the output signal. For comparison we may note that power electronic switches typically operate at frequencies between 2 and 100 kHz. As a consequence, the ratio m between the switching and reference frequencies, the so-called frequency modulation ratio, typically takes values in the interval from 40 to 2000 (for simplicity, we consider this value as an integer number). A discrete time model of the inverter must capture its dynamics after one full period of the output voltage. Hence, in addition to the map f, which describes the dynamics after one switching step, we have also to consider the mth iterate f^m of the map f. For large values of m, this map by construction has a very high number N of switching manifolds, and this number varies with the parameters of the model (see Fig. 1(a)). To our knowledge, except for direct calculation, no method is presently available that can estimate the exact number of N for a given set of parameters, and only an upper limit can be provided. However, already in the simplest case, when the map f only has two switching manifolds, the upper limit to N is $N_{\max} = 3^m$. Although this upper limit is typically not reached, it can be stated that such models have an extremely high number of switching manifolds and belong therefore to a class of systems which have never been investigated before.

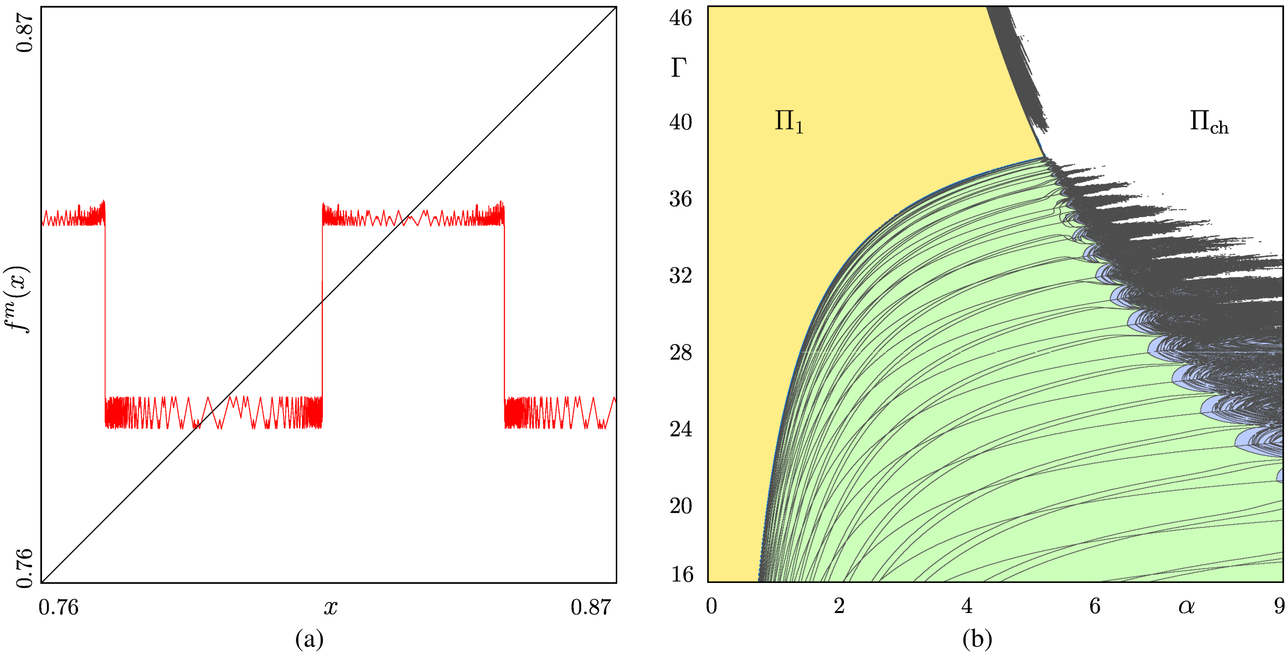

Figure 1: (a) Typical shape of the map f^m with $m = 100$. This map depicts the dynamics of a single phase H-bridge inverter with bipolar pulse-width modulation over one period of the reference signal. (b) Structure of the (α, Γ) parameter plane for the considered model. The parameters α and Γ represent the corrector gain factor and the normalized input voltage, respectively. In the domain Π_1 the map f^m displays a globally attracting fixed point. Curves inside this domain indicate persistence border collisions. The domain Π_{ch} represents the region of chaotic dynamics.

Bifurcation structures

This work presents the results of an investigation of the dynamics of single phase H-bridge inverters with unipolar and bipolar pulse-width modulated control. We have shown that:

Transitions to chaos via irregular cascades of border collisions: The boundary between the domain Π_1 associated with stable, globally attracting fixed points of f^m and the domain Π_{ch} that represents a region of undesired high frequency and low amplitude chaotic oscillations may display an unusually complicated oscillating shape (Fig. 1(b). The transition to chaos in these models occurs via irregular cascades of different border collisions, some of which lead to bifurcations while others do not. The latter type bifurcations, in which transitions of fixed points across switching manifolds without change of stability occurs, are referred to as persistence border collisions, [1].

Structures inside the stability domain of fixed points: As illustrated by Fig.1(a), the domain Π_1 has a complicated interior structure formed by boundaries defined by persistence border collisions. By following these boundaries, we could identify regions in parameter space leading to qualitatively different output signals even though they were all associated with globally attracting fixed points of the corresponding stroboscopic map. Investigating the interior structure of one such region, we explained also the shape of the boundary between Π_1 and Π_{ch}, [3].

Alignment of smooth bifurcations: The bifurcation structures observed in the considered class of models include not only border-collision related phenomena, but also involve classical smooth (pitchfork and flip) bifurcations. We have shown that different fixed points of the stroboscopic map f^m may (under certain conditions) undergo these smooth bifurcations in such a way that a macroscopic pattern stretching across the overall bifurcation structure is formed [4].

Undetectable parts of chaotic attractors: In addition to the unusual phenomena described above, our models display a peculiarity related to the properties of the chaotic attractors. We have noticed that these attractors can include subsets with extremely low density, a density which in some cases falls well below the values that are numerically computable by standard methods. We show that this phenomenon is caused by the extremely small size of the partitions mapped by f^m to the corresponding parts of attractors.

References

[1] V. Avrutin, E. Mosekilde, Zh.T. Zhusubaliyev, and L. Gardini. Onset of chaos in a single-phase power electronic inverter. *Chaos*, 25:043114, 2015.

[2] V. Avrutin, Zh.T. Zhusubaliyev, A. El Aroudi, D. Fournier-Prunaret, G. Garcia, and E. Mosekilde. Disrupted bandcount doubling in an AC-DC boost PFC circuit modeled by a time varying map. *J. of Physics*, 692(1):012003, 2016.

[3] V. Avrutin, Zh.T. Zhusubaliyev, and E. Mosekilde. Border collisions inside the stability domain of a fixed point. *Physica D*, 321-322:1–15, 2016.

[4] V. Avrutin, Zh.T. Zhusubaliyev, and E. Mosekilde. Cascades of alternating pitchfork and flip bifurcations in H-bridge inverters. *Physica D (to appear)*, 2017.