Selected aspects involved in dynamics and optimization of cranes with a pivoting boom

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Abstract

Cranes with a pivoting jib are complex dynamic systems governed by nonlinear, non-stationary differential equations of motion [4, 5]. Certain crane operations: hoisting/lowering the payload connected with a slewing jib require a nonlinear description to take into account of Euler and Coriolis forces whose impacts should be minimized already at the stage of selection of the system parameters and mechanism structure. Thus obtained optimal sets of parameters for the above-mentioned mechanisms were optimized for the full range of the slewing motion. It is demonstrated that selection of geometric dimensions of structural elements of the hoisting mechanisms, i.e. the slewing system and counterbalances enables the horizontal track error load to be minimized whilst the forces acting on the mechanism and inducing its vertical movement can be reduced. Thus for the assumed lifting capacity and distance jaunt we get the structure of the crane mechanism that guarantees the minimal energy consumption. This study investigates the energy efficiency of the jib lift mechanism structures: that with unilateral constraints (rope mechanisms) and with bilateral constraints (eg. lever mechanisms), so that they can be optimized together with the jib-balancing mechanism.

1. Introduction

Previous works on this subject [2, 3] were limited in scope as the analysis was mostly restricted to a few selected boom positions. More recent works such as [6, 7] focused on the search for the optimal position of blocks in a compensation mechanism such that the boom's unbalance moment should be minimized. In the work [6] a minimum deviation of the vertical load is sought for a finite number of boom positions, basing on the linearized form of the objective function. A similar problem to is investigated in [7] with respect to the two rocker port crane.

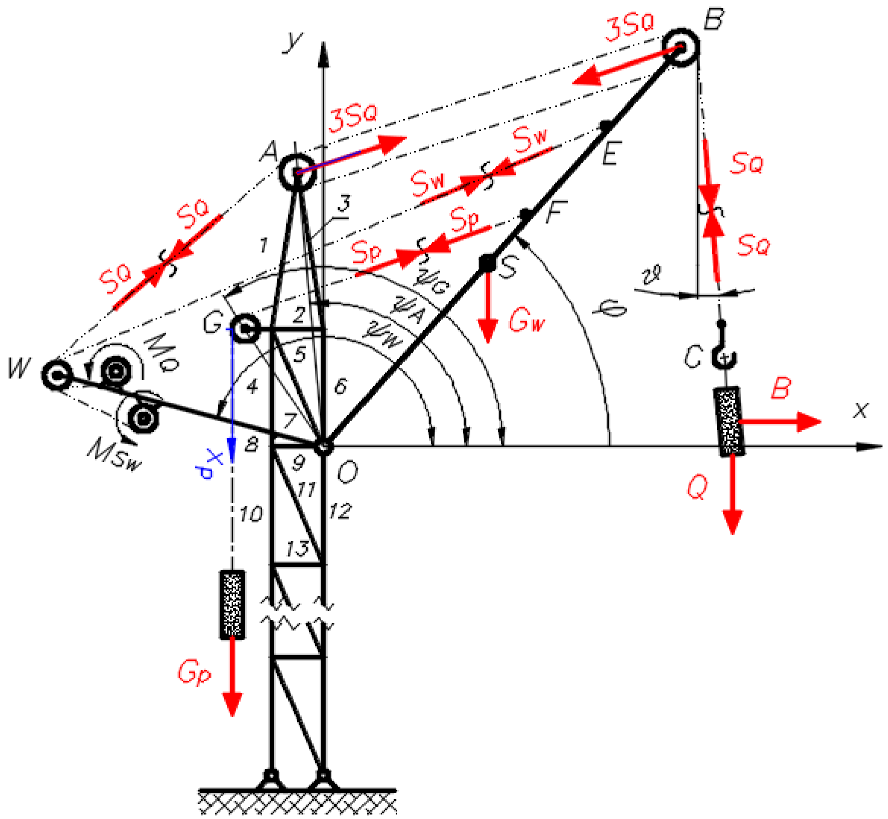

Fig. 1. Physical model of a one-link crane

The study investigates the behavior of a crane with a pivoting jib, whose physical model is shown in Fig. 1, subjected to the applied loads: Q – lifting load due to hoisted mass, G_P – counterweight and G_W – weight of the jib. Respective forces acting in ropes due to lifting load – S_Q, jib lifting – S_W, counterweight – S_P act at acute angles to the jib: α, β, γ - not indicated in Figure 1. The physical model of a one-link crane is governed by the following equations of motion:

$$\begin{cases} J_{WO}\varepsilon_{\varphi} = L_{OB}\left[3\cdot sin(\alpha)\cdot S_Q - cos(\varphi-\upsilon)\cdot S_Q + \kappa_{OE}\cdot sin(\beta)\cdot S_W + \kappa_{OF}\cdot sin(\gamma)\cdot S_P - \kappa_{OS}\cdot cos(\varphi)\cdot G_W\right] \\ m_Q\left[a_{BC} - L_{BC}\cdot\omega_{\upsilon}^2 - L_{OB}\,sin(\varphi-\upsilon)\cdot\varepsilon_{\varphi} + L_{OB}\,cos(\varphi-\upsilon)\cdot\omega_{\varphi}^2\right] = cos(\upsilon)\cdot Q - S_Q \\ m_Q\left[L_{BC}\cdot\varepsilon_{\upsilon} + 2\cdot\omega_{\upsilon}\cdot v_{BC} - L_{OB}\,cos(\varphi-\upsilon)\cdot\varepsilon_{\varphi} - L_{OB}\,sin(\varphi-\upsilon)\cdot\omega_{\varphi}^2\right] = -sin(\upsilon)\cdot Q \\ m_P a_{XP} = G_P - S_P \end{cases} \tag{1}$$

where: ε_φ, ω_φ – angular acceleration and angular velocity of the jib, ε_υ, ω_υ – angular acceleration and angular velocity of the load Q, a_{BC}, v_{BC} – acceleration and velocity of the longitudinal motion of the load Q, a_{XP} – vertical acceleration of the counterweight, L_{OB} – length of the jib, κ_{OE}, κ_{OF}, κ_{OS} – normalized with respect to L_{OB} distances: $\overline{OE}, \overline{OP}, \overline{OS}$.

Optimization of mechanical structures such as to minimize the operating dynamic forces and maximum energy uptake is categorized as an vibration isolation method, involving the reduction of the energy of the vibration source.

For each problem involving the dynamic behavior of investigated crane mechanisms, the specific optimization task is formulated by defining the objective function:

1) slewing mechanism → minimizing the change of the payload level position – Δy, whilst the winch is immobile,
2) counter weight → minimizing the jib lifting work,
3) jib lifting mechanism → minimum force in the rope winch jib.

2. Parametric optimization of a slewing mechanism

The first step in optimization of mechanisms in a crane with a pivoting jib should involve the slewing mechanism. The optimization procedure will determine the position of the peak pulley whilst the objective function is minimization (reduction) of the horizontal track error. The change of the slewing range is implemented by the slewing mechanism. It is vital that the slewing motion should not bring about the change of the payload level position.

It is assumed that the hoisting load winch mechanism is blocked, hence the rope length L on which the payload is suspended will not change during the hoisting phase.

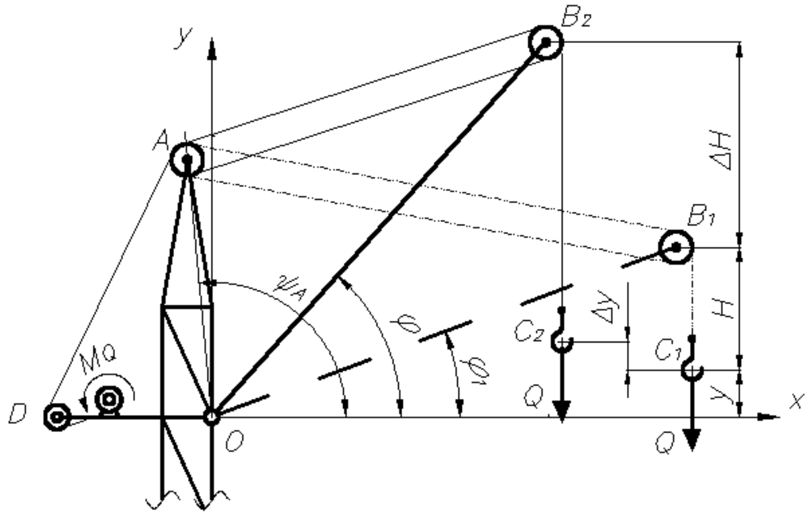

Fig. 2. Change of the jib's angular position in a one-link crane with a blocked winch

In the case of extensible jibs this condition can never be fully satisfied. The design of the slewing mechanism is considered satisfactory if the horizontal track error load during the slewing – δ is less than 2%. The loads horizontal track error is understood as the absolute value of the ratio of the payload deviation from the straight-line trajectory to the total change of horizontal deviation:

$$\delta = \left|\frac{y(\varphi_{min}) - y(\varphi_{max})}{x(\varphi_{min}) - x(\varphi_{max})}\right|\cdot 100\% \ , \tag{2}$$

where: the angles φ_{max} and φ_{min} correspond to the lowest and the highest position of the jib, respectively.

Two configurations of the slewing mechanism represented by the jib's inclination angles φ_1, φ_2 are shown in Fig. 2. Hence we write:

$$\begin{cases} i_w \cdot \overline{AB_1} + \overline{B_1C_1} = L \\ i_w \cdot \overline{AB_2} + \overline{B_2C_2} = L \\ L = const \end{cases} \tag{3}$$

The change of the jib's angular position involves a change of distance between the axes of the rope pulley *B* and the top pulley *A*, thus changing the length of the rope's free end section ΔL_{BC} on which the payload is suspended. On the other hand, when the pulley dimensions are omitted as negligibly small in relation to the distance between them, then ΔLBC can be derived from the formula:

$$\Delta L_{BC} = \Delta H - \Delta y = i_w\left(L_{AB1} - L_{AB2}\right), \tag{4}$$

where: i_w – transmission ratio of the compensating pulley block, L_{ABi} – distance between the rope pulleys *A* and *B* for an arbitrary *i*-th position of the jib (ie $L_{ABi} = \overline{AB_i}$).
Comparing two arbitrary angular positions of the jib we get the formula expressing the payload height:

$$\begin{cases} y_1 = L_{OB} \cdot sin(\varphi_1) - H \\ y_2 = L_{OB} \cdot sin(\varphi_2) - H - (\Delta H - \Delta y) \end{cases}. \tag{5}$$

The change of the payload position level Δy caused by varying the jib's angular position $\varphi_1 \to \varphi_2$ is given as:

$$\Delta y(\varphi) = L_{OB} \cdot sin(\varphi) - L_{OB} \cdot sin(\varphi_1) - i_w\left[L_{AB}(\varphi_1) - L_{AB}(\varphi)\right], \tag{6}$$

and: $L_{AB}(\varphi) = \sqrt{L_{OA}^2 + L_{OB}^2 - 2L_{OA}L_{OB} \cdot cos(\psi_A - \phi)}$, (7)

where: L_{OA} – distance between the rotation axis of the jib *O* and the top pulley *A*, ψ_A – angle of horizontal inclination of the line connecting the rotation axes of the jib *O* and the top pulley *A* (typically $\pi/3 \le \psi_A \le 2\pi/3$).
For a stabilized angular position of the jib φ_1, we get:

$$\Delta y(\varphi) = L_{OB}\left[sin(\varphi) - sin(\varphi_1)\right] - i_C\left[\sqrt{\kappa_{OA} + 1 - 2\kappa_{OA} cos(\psi_A - \varphi)} - \sqrt{k_A + 1 - 2\kappa_{OA} cos(\psi_A - \varphi_1)}\right] \tag{8}$$

where: $\kappa_{OA} = \dfrac{L_{OA}}{L_{OB}}$. (9)

In Fig. 1 the hook is attached directly to the rope's end. In most cranes an additional pulley block is connected between the jib top incorporating the pulley *B* and the hook, thus forming a sheave block with the transmission ratio $i_z > 1$. In this situation the transmission ratio of the entire slewing mechanism becomes $i_c = i_w / i_z$, where $i_{c\,min} = 3$ [2, 3].
Thus formulated optimization problem uses an objective function Δy, the decision variables being κ_{OA} and the angle ψ_A. The length L_{OB} w and the transmission i_w (i_c) are taken as constant in the optimization procedure. It is assumed that L_{OB} = 30 m, the angle variability range $\varphi \in [15° \div 75°]$.

Optimization problem 1

The optimization problem involves finding the optimal values of κ_{OA} and the angle ψ_A for which the quadratic functional $J(\kappa_{OA}, \psi_A)$ reaches its minimum, assuming that $\varphi_1 = \varphi_{min}$.

$$J(\kappa_{OA}, \psi_A) = \int_{\varphi_{min}}^{\varphi_{max}} \left[\Delta y(\varphi, \kappa_{OA}, \psi_A)\right]^2 d\varphi . \tag{10}$$

The adopted criterion is important, yet still insufficient. One has to bear in mind that it is crucial that the derivative $dy/d\varphi$ be minimized, since it determines the inertia forces acting upon the slewing mechanism during the hoisting hoisting/lowering the jib. The weight *R* being ascribed to the function Δy, and the function $dy/d\varphi$ added with its ascribed weight *P*, we get a new optimization criterion, combining the two previous ones [1].

$$J(\kappa_{OA},\psi_A)=\int_{\varphi_{min}}^{\varphi_{max}}\left\{P\cdot\left[\frac{dy(\varphi,\kappa_{OA},\psi_A)}{d\varphi}\right]^2+R\cdot[\Delta y(\varphi,\ \kappa_{OA},\psi_A)]^2\right\}d\varphi. \tag{11}$$

Restrictions imposed on the fixed jib length L_{OB}:

$$\begin{cases} i_w \geq 3 \quad where:\ i_w = 2n+1, \quad and \quad n = 1,2,..,\\ \frac{\pi}{3}\leq \psi_A \leq \frac{2\pi}{3},\\ 0\leq k_A \leq \frac{1}{2}. \end{cases} \tag{12}$$

When the functional (10) is replaced by (11) in the optimization problem 1 for the stable conditions (12), we get new solutions for the parameters κ_{OA}, ψ_A. It appears, that extending the optimization criterion to incorporate the condition imposed upon the derivative $dy/d\varphi$ leads to minimization of the inertia force and further, reduces the deviation from the straight-line trajectory in the payload's motion in relation to the solution (10) [1, 2, 3].

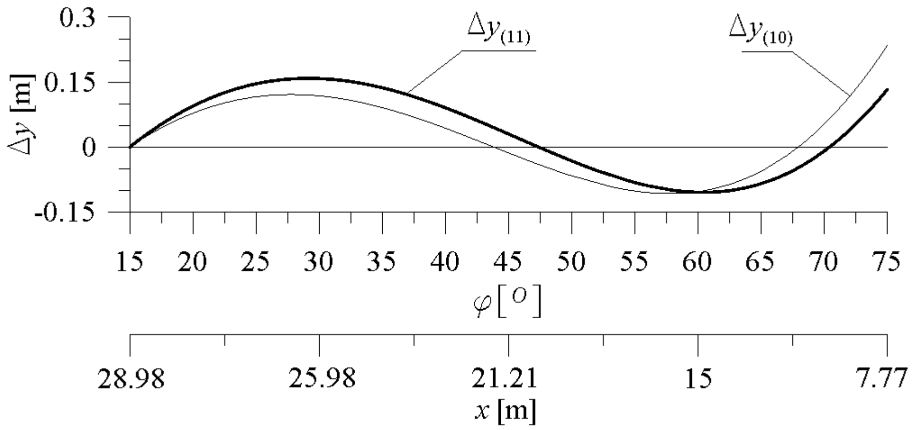

Fig. 3. Hook's trajectory during the radius change over its full range obtained for two criterions: (10) and (12)

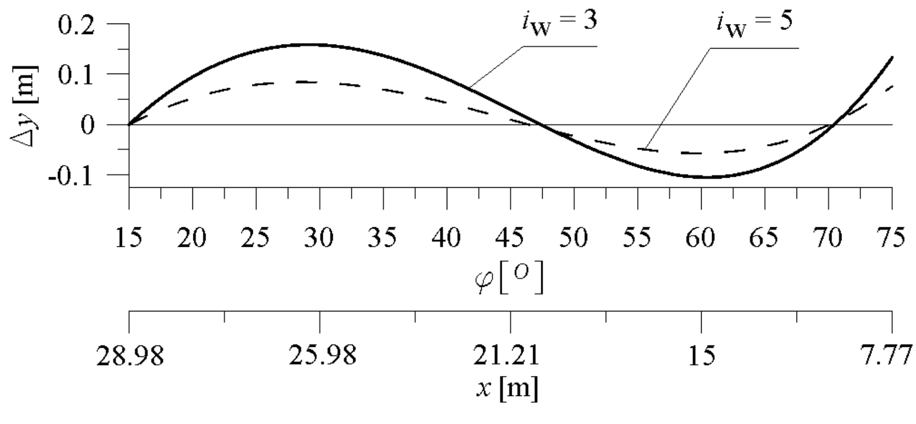

Fig. 4. Hook's trajectory during the radius change over its full range for two values of the transmission ratio of lifting mechanism $i_w = 3$ and $i_w = 5$

For two values of transmission of the compensating rope system: $i_w = 3$ and $i_w = 5$ for the fixed values of weight coefficients $P = 1$ and $R = 1$ in functional (12), we get:

$$i_w = 3 \rightarrow \begin{cases}\kappa_{OA} = 0.3078\\ \psi_A = 83.2674^\circ\end{cases}, \quad i_w = 5 \rightarrow \begin{cases}\kappa_{OA} = 0.1901\\ \psi_A = 85.9712^\circ\end{cases} \tag{13}$$

The deviation from the straight-line trajectory for becomes $i_w = 3 \rightarrow \delta_3 = 1.242$ %, and for $i_w = 5 \rightarrow \delta = 0.666$ %. Fig. 3 shows the trajectory of the hook when the crane radius changes over its entire range, for each pair of solutions (13).

When the transmission ratio of the jib lifting mechanism is increased, the payload's horizontal trajectory better approximates the straight-line motion; however the rope resistance due to winding is increased, too.

3. Parametric optimization of the jib balance system

Balancing of the jib in a one-link crane requires the selection of the jib ballast weight and position of the pulley, with respect to the jib's rotation axis such as to minimize the work required for a slewing change. Figure 1 shows the loads acting on the jib in a one-link crane. Recalling the previous optimization problem, the following designations are adopted: L_{OG} – distance between the rotation axis of the jib O and the pulley G, ψ_G – angle of horizontal inclination of the line segment $\overline{OG}$. The residual unbalance moment of the jib is a function of the angular position φ:

$$M(\varphi) = (G_W L_{OS} + Q L_{OB})\cos(\varphi) - G_P L_{OF} \frac{L_{OG} \sin(\psi_G - \varphi)}{L_{GF}(\varphi)} - 3 Q L_{OB} \frac{L_{OA} \sin(\psi_A - \varphi)}{L_{AB}(\varphi)}. \tag{14}$$

where: $$L_{GF}(\varphi) = \sqrt{L_{OF}^2 + L_{OG}^2 - 2 L_{OF} L_{OA} \cdot \cos(\psi_G - \phi)}. \tag{15}$$

Optimization problem 2

In this optimization task $M(\varphi, L_{OG}, \psi_G, L_{OF}, G_P)$, becomes the objective function and the decision variables are: position of the pulley G defined by distance – L_{OG} and angle – ψ_G, distance from the axis of the pin jib – O to the point where the rope counterweight is attached to the arm of – L_{OF}, weight of counterweight – G_P. Distances: L_{OA} , L_{OB} , L_{OS} and weight Q and G_W are constant parameters in the optimization procedure. For counterweight configurations as in Figure 1, where the rope is connected to the jib creating a mechanism with unilateral bonds, the jib is balanced without the payload Q. The task consists in determination of the optimal values: L_{OG}, L_{OF}, ψ_G, G_P, for which the quadratic functional $J(L_{OG}, L_{OF}, \psi_G, G_P)$, reaches a minimum.

$$J(L_{OG}, \psi_G, L_{OF}, G_P) = \int_{\varphi_{min}}^{\varphi_{max}} \left[M(\varphi, L_{OG}, \psi_G, L_{OF}, G_P)\right]^2 d\varphi. \tag{16}$$

$$\text{Assumed condition:} \begin{cases} \psi_A, L_{OA}, L_{OB}, L_{OS}, G_W, Q\,(Q=0) - \text{known parameters}, \\ M(\varphi, L_{OG}, \psi_G, L_{OF}, G_P) > 0 \ \text{ for each } \ \varphi_{min} \le \varphi \le \varphi_{max}, \\ 0.5 G_W \le G_P \le 1.7 G_W \\ 0 < L_{OG} \le \frac{1}{2} L_{OB} \\ 0 < L_{OF} \le L_{OB} \\ \frac{\pi}{3} \le \psi_G \le \frac{2}{3}\pi \end{cases} \tag{17}$$

The solution to the optimization task No. 2, for G_w = 45 kN, L_{OS} = 12.857 m, L_{OA} = 9.234 m, ψ_A = 83.2674º, is the set of parameter values such that the functional (16) is minimized for the imposed constraint conditions (18):

$$\left.\begin{matrix} L_{OG} = 7.0605\,\text{m} \\ L_{OF} = 30\,\text{m} \\ \psi_G = 85.489^\circ \\ G_P = 67\,\text{kN} \end{matrix}\right\} \rightarrow L_\varphi = \int_{\varphi_{min}}^{\varphi_{max}} M(\varphi, L_{OG}, \psi_G, L_{OF}, G_P)\, d\varphi = 51.43\,\text{kJ} \tag{18}$$

It appears that the best solution is obtained when the counterbalance rope is attached to the end of the jib, ie. when $L_{OF} = L_{OB}$. Optimization of the rope mechanism in the counterweight rope is discussed in more detail in [1] and the assumption that the counterbalance rope is attached to the tip of the jib adopted is based on a review of the existing crane design options. From the standpoint of mathematics, the solution (18) confirms the validity of this assumption. Polar coordinates of the pulley mechanism in a counterweight are – G [7.0605 m, 85.489º].

The effectiveness of the parametric optimization of the movable counterweight mechanism is expressed as work – L_φ needed to change the jib's angular position over the entire variability range of its inclination angle, the inertia and friction forces being neglected.

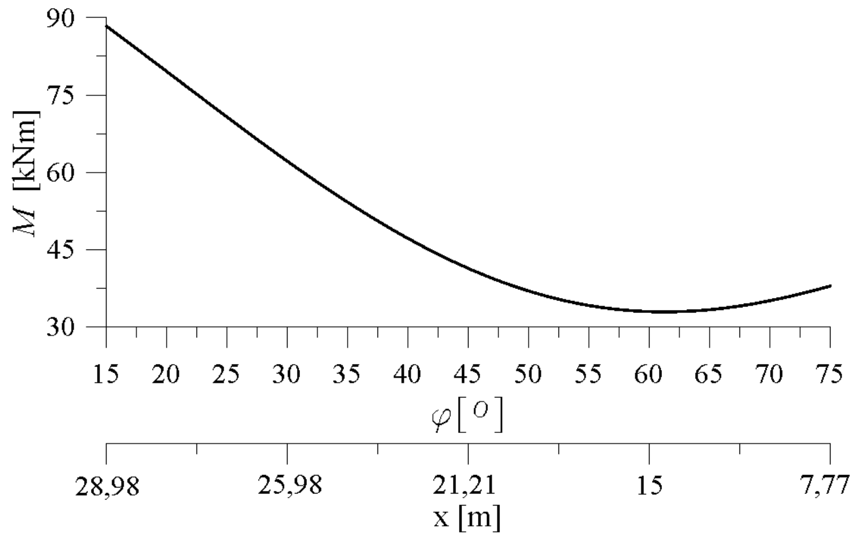

Fig. 5. Residual moment of jib unbalance in the function of the radius change

Under thus defined conditions, this quantity is expressed as the integral (18) and equals 51.43 kJ. Finally, it is recommended that the counter jib ballast weight should be taken 10% less than, the value predicted in the optimization problem to make up for resistance due to friction when the jib is lowered. This recommendation applies only to unilaterally constrained mechanisms. When a mechanism with bilateral constraints is considered, for example a lever mechanism in a four-bar linkage the positive residual torque requirement could be eliminated. The jib slewing work could be effectively reduced through optimizing the structural design of such mechanism.

4. Parametric optimization of the jib lifting mechanism

Optimization of slewing mechanism discussed in section 1 in fact could be applied to the rope mechanism in a winch. The main objective was to ensure such roping configuration so as to minimize the horizontal hook trajectory error of for the full variability range of the change in the jib's angle of horizontal inclination when the winch is blocked. Forces required to lift the jib have not been considered so far. Recalling (14), the force acting in the rope lifting the jib can be written as:

$$S_W(\varphi)=\frac{L_{EW}(\varphi)}{L_{OE}L_{OW}\,sin(\psi_W-\varphi)}M(\varphi), \tag{19}$$

where: $L_{EW}(\varphi)=\sqrt{L_{OW}^2+L_{OE}^2-2L_{OW}L_{OE}\cdot cos(\psi_W-\phi)}$. (20)

Optimization problem 3

The optimization problem involving the jib lifting mechanism consists in finding the point where the rope is attached to the jib, as well as polar coordinates the pulley axes – $W\,[L_{OW},\psi_W]$ associated with the tower crane, for which the force in the winch will be the lowest in terms of rms value and should be positive. All parameters determined in earlier sections remain constant in throughout the optimization of lifting the jib mechanism.

$$J(L_{OE},L_{OW},\psi_W)=\int_{\varphi_{min}}^{\varphi_{max}}[S_W(\varphi,L_{OE},L_{OW},\psi_W)]^2\,d\varphi, \tag{21}$$

$$\text{Assumed condition}:\begin{cases}\psi_A,\psi_G,\mathrm{L_{OA}},\mathrm{L_{OB}},\mathrm{L_{OS}},\mathrm{L_{OA}},\mathrm{L_{OB}},G_w,Q,G_p-\ \text{known parameters,}\\ S_W(\varphi,L_{OE},L_{OW},\psi_W,)>0\quad \text{for each } \varphi_{min}\le\varphi\le\varphi_{max},\\ 0<L_{OE}\le L_{OB}\\ 0<L_{OW}\le\frac{1}{3}L_{OB}\\ \frac{\pi}{3}\le\psi_W\le\pi\end{cases} \tag{22}$$

The solution to the optimization task No. 3 for the nominal value of load Q = 50 kN is the following set of parameter values that minimize the functional (21), under the imposed conditions (22):

$$\left.\begin{aligned} L_{OE} &= 30m \\ L_{OW} &= 10\,m \\ \psi_W &= 116.4911° \end{aligned}\right\} \tag{23}$$

The best solution is obtained when the rope is attached to the end of the jib, that is when $L_{OE} = L_{OB}$.
Polar coordinates of the pulley axes W [10 m, 116.4911°]. The torque required to hoist the jib expressed in terms of (14) is not dependent on parameters of the jib lifting mechanism and neither is work required to lift the jib.
For previously determined parameters of the slewing and counterweight mechanisms, and under the loading conditions due to the nominal payload – Q and the weight of the jib G_W operating at the distance – L_{OS} from the axis of the pin jib, the hoisting work becomes:

$$L_\varphi = \int_{\varphi_{min}}^{\varphi_{max}} M(\varphi)\, d\varphi = 58.07 \text{ kJ} \tag{24}$$

Optimization of the jib lifting mechanism results in reduction and balancing of forces in the rope over the entire range of angle variability $\varphi \in [15° \div 75°]$.

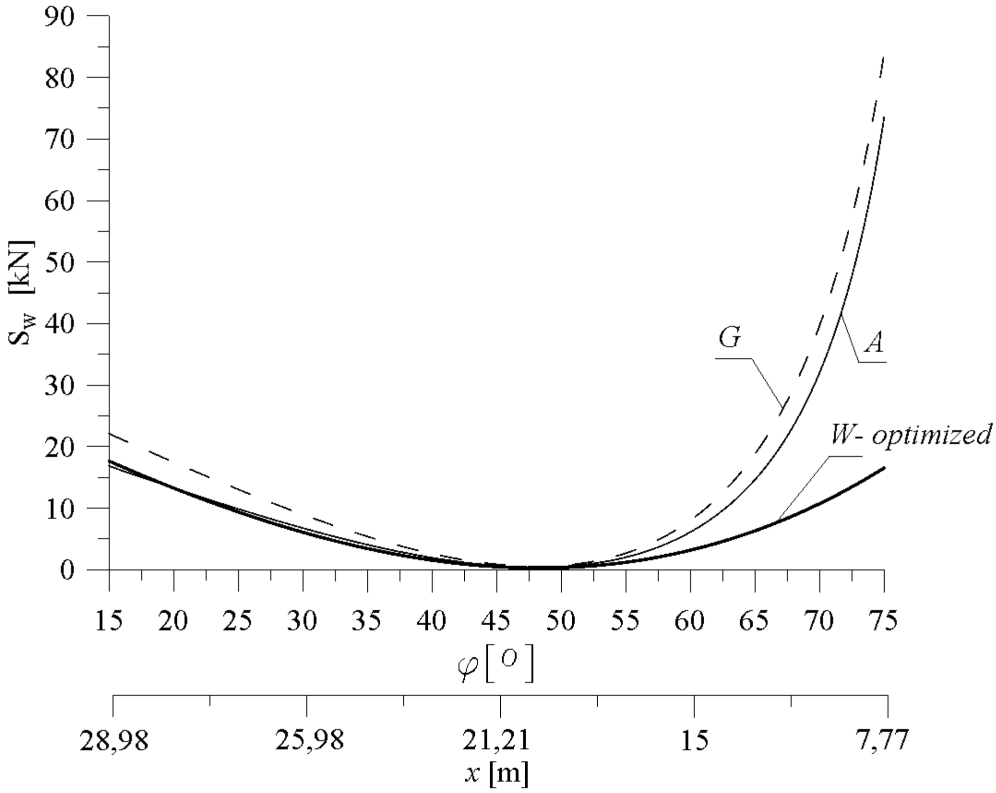

Fig. 6. Controlling forces acting in the rope through selection of the rotation axis of the pulley in the jib-lifting mechanism (plot designations correspond to points A, G, W in Figure 1)

In Fig. 6 the forces are compared that act in the rope hooked on the movable end of the jib and which runs through a pulley located at the point A or G or W, depending on the design option, in accordance with designations shown in Fig. 1. Three plots of force acting in the in the jib winch rope are derived and their common feature is the minimum value achieved for the slewing angle – $\varphi \approx 50$ °.
The least favorable force variability pattern was obtained when the axis of the pulley in the jib lifting mechanism coincides that in the pulley of counterweight mechanism – G. The values of force acting in the rope – S_W decrease from 22.1 kN to nearly zero then rise again to achieve the maximum angle of deception 84.0 kN (dashed line – G in Fig. 5).

The variability pattern of the force acting in the rope was achieved when the axis of the pulley in the jib lifting mechanism coincides with that of the pulley in the winch mechanism – *A*. The force value S_W decreases from 16.85 kN to nearly zero and then rises again to 73.6 kN for a maximum value of angle φ, (thin line – *A* in fig. 5).
The most favorable pattern of force acting upon the rope, is obtained when the axis of rotation of the pulley in the jib lifting mechanism is at the point – *W*. The values of force S_W goes down from 17.67 kN to nearly zero and then rises again approaching 16.56 kN for the maximum value of the angle φ (thick line – *W* in Fig. 5).
Advantages of minimizing the force acting in the rope in the jib lifting mechanism are:

- Small rope diameter → small pulley → low resistance during rope winding,
- Low-power electric motors (approximately 7 kW) → reduced energy demand,
- Small force variations in ropes → less overloading of electric motors → little overheating of engines.

Because of the unilateral constraints it is recommended in the optimization process that the jib weight should be taken 10% less than in real life conditions.

5. Concluding remarks

Optimization tasks involving the three rope mechanisms in a one-link jib crane lead us to the following conclusions:

1) Application of dedicated software (such as Mathcad) to solve variational problems such as finding a minimum of properly formulated quadratic functionals proves to be very effective and rapid solution to parametric optimization problems.
2) Even though functionals (9), (10), (14), (19) are formally quadratic, it is not required that the Riccati equations be solved.
3) When the optimization criterion for the slewing mechanism is extended to incorporate the condition imposed on the derivative $dy/d\varphi$, the form of the quadratic functional (9) becomes more complicated but the numerical solution still can be found.
4) The optimization effectiveness of the boom luffing mechanism determines the level of vibration reduction of the cargo hanged on the hook.
5) The optimization problem is solved and solutions are obtained in the form of set of mechanism parameters for which the work involved in payload hoisting should be minimal. The force acting in line in the jib lifting mechanism should be minimal.
6) For the assumed lifting capacity and distance jaunt we get the structure of the crane mechanism that guarantees the minimal energy consumption.

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