Non-stationary attractors in forced and damped weakly coupled pendulums

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Summary. For searching the attractors of the weakly-coupled damped and forced pendulums we used fundamental stationary and non-stationary solutions of corresponding conservative system which are NNMs and LPTs. The conditions in parametric space determining the existence of stable resonance oscillations under action of dissipative and external periodic forces have been found from the balance of their works on NNMs and LPTs. It was shown for the first time that an attractor can be realized not only on NNM (cooperative or localized), but on the LPT also. It means that the stable nonlinear beats between two pendulums can arise. It was seen that such regime can exist in the case of sufficiently large amplitudes, and therefore can not be revealed in the conventional quasi-linear approximation. The obtained results are confirmed by computer simulation data.

Damped and forced coupled nonlinear oscillators are very significant models in Nonlinear Dynamics, and have various applications in classical physics and quantum mechanics [1, 2]. In classical physics, such discrete models arise either due to a weak coupling between relatively stiff elements of the system or as a result of a continuous model discretization. In quantum theory, such models describe the two-contact Josephson interferometer or the chains of Josephson junctions. A theoretical basis for studying the non-stationary dynamics is provided by the concept of Limiting Phase Trajectories (LPTs) [3]. Basic stationary regimes, Nonlinear Normal Modes (NNMs), are not involved in the processes of energy exchange. Contrary to this, the LPTs describe the maximum possible (under the given conditions) periodic energy exchange between the oscillators or clusters of oscillators (coherent domains - CDs) in a slow time scale [4, 5]. It is well known that modal instability is accompanied by the appearance of localized NNMs with energy domination in one of the coherent domains (stationary localization). On the contrary, the LPT instability leads to the impossibility of the intensive energy exchange between CDs and to the localization of energy on the initially excited CD [4, 5]. We consider strongly nonlinear processes without any restrictions on the amplitudes of oscillations. To overcome the principal difficulties caused by the absence of a small parameter at the nonlinear terms in the initial equations of motion, we use a self-consistent semi-inverse procedure [6]. The origin and magnitude of the small parameter are determined in the process of analysis. The mathematical formulation of the problem turns out to be equivalent to the description of two coupled damped pendulums under a periodic external force at arbitrary amplitudes of the pendulums [6, 7].
Weak coupling between the pendulums leads to the manifestation of slow (comparing to the force period) time scale and allows to successfully apply the two-scale asymptotic procedure. This procedure is applied to the analysis of the equations of motion in terms of complex variables, which are the classic counterpart of the creation and annihilation operators in the quantum theory. As a result, we have obtained the main asymptotic approximation in the slow time as series by the coupling parameter.
To find the attractors of this system, we used the fundamental stationary and non-stationary solutions of corresponding conservative system. These solutions are NNMs and LPTs. Conditions in the parametric space determining the existence of stable resonance oscillations under action of dissipative and external periodic forces have been found from the balance of their works on NNMs and LPTs. It was shown for the first time that an attractor can be realized not only on NNM (cooperative or localized), but on the LPT also.

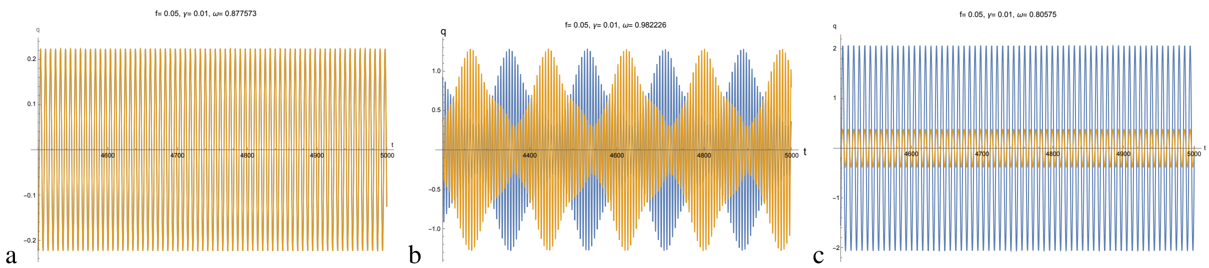

Figure 1: Attractors on the NNM and LPT (a-b), and transition to energy localization (c)

This means that the stable nonlinear beats between the two pendulums can arise. It was seen that such regime can exist in the case of sufficiently large amplitudes, and therefore can not be revealed in the conventional quasi-linear approximation. The obtained results are confirmed by computer simulation data. We also discuss an extension of these results on the case of a finite pendulum chain.
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References

[1] Braun O., Kivshar Yu. (2004) The Frenkel-Kontorova model. Concepts, Methods, and Applications. Springer-Verlag Berlin Heidelberg.

[2] Cuevas-Maraver J., Kevrekidis P. G., Williams F. (2014) The sine-Gordon Model and its Applications: From Pendula and Josephson Junctions to Gravity and High-Energy Physics. Springer International Publishing, Switzerland.

[3] Manevitch L.I. (2007) New approach to beating phenomenon in coupled nonlinear oscillatory chains *Arch. Appl. Mech.* **77**, 301.

[4] Manevitch L.I., Smirnov V.V. (2010) Limiting phase trajectories and the origin of energy localization in nonlinear oscillatory chains *Phys Rev E* **82**, 036602.

[5] Smirnov V.V., Manevitch L.I. (2011), Limiting Phase Trajectories and Dynamic Transitions in Nonlinear Periodic Systems. *Acoustical Physics*, **57**, 271-276.

[6] Manevitch L.I., Smirnov V.V. (2016) Semi-inverse method in the nonlinear dynamics *Proceedings of the 5th International Conference on Nonlinear Dynamics* 28-37

[7] Manevitch L.I., Smirnov V.V., Romeo F. (2016) Non-stationary resonance dynamics of the harmonically forced pendulum *Cybernetics and Physics* **5**(3), 91-95.