Applications of Spectral Submanifolds in Nonlinear Modal Analysis

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Summary: For dissipative oscillatory systems, a spectral submanifold (SSM) is the smoothest invariant manifold among those asymptotic to a modal subspace of a generalized nonlinear normal mode (NNM). SSMs offer a mathematically exact foundation for constructing reduced-order models. Furthermore, computing the dynamics restricted to the SSMs yields backbone curves that characterize the exact nature of modal nonlinearities. In this talk, we discuss several applications of these recent ideas to multi-degree-of-freedom mechanical systems.

Keywords: spectral submanifolds, model reduction, nonlinear normal modes, structural dynamics, forced vibrations

Background

An inherent feature of nonlinear multi-degree-of-freedom mechanical systems is the complex interaction they exhibit among different degrees of freedom. In real-life applications, nonlinearities are ubiquitous, increasing the complexity of the systems behavior. One often seeks reduced-order models that are able to capture this complexity in an accurate manner with reduced numerical effort in simulations. Model-order reduction methods for linear vibrational systems often make use of the modal superposition principle, where the orthogonality of the linear normal modes can be exploited to decouple the governing equations of motion.

In decomposing nonlinear oscillations, a fundamental notion is the nonlinear normal mode (NNM) concept of Rosenberg [1]. He defines a nonlinear normal mode as a synchronous periodic oscillation that reaches its maximum in all modal coordinates at the same time. An alternative definition, proposed by Shaw and Pierre [2], is a nonlinear continuation of the subspaces of linear normal modes into invariant manifolds that are locally graphs over those subspaces. A clear relationship between these two views on NNMs is established for conservative systems by [3] and [4]. These references guarantee (under appropriate non-resonance conditions) the existence of a unique, analytic and robust Shaw—Pierre-type invariant manifold tangent to any two-dimensional modal subspace of the linearized system. This manifold, in turn, is filled with Rosenberg-type periodic orbits.

In a non-conservative setting, a unified approach has been proposed by [5] to clarify the relationship between the two NNM concepts. Specifically, [5] defines a spectral submanifold (SSM) as the smoothest invariant manifold tangent to a modal subspace of a generalized NNM, with the latter including any type of recurrent motion with finitely many harmonics. The existence, uniqueness and persistence results of autonomous and non-autonomous SSMs provide an exact mathematical foundation for constructing nonlinear reduced-order models over appropriately chosen spectral subspaces. These models are obtained by reducing the full dynamics to the exactly invariant SSM surfaces.

Summary of results

In this talk, we discuss several applications of SSMs to multi-degree-of-freedom mechanical systems. The SSMs can either be constructed as graphs over a chosen spectral subspace in the phase space, as in the Shaw-Pierre approach, or as an embedding of the spectral subspace in the phase space itself. The latter approach is known as the parameterization method, developed first in [6]. As an advantage, the parameterized construction of SSMs does not break down when the SSM folds over the underlying spectral subspace. Amplitude-frequency plots of the dynamics restricted to the SSMs are the exact mathematical realizations of backbone curves, which are frequently sought in experimental nonlinear model identification to characterize modal nonlinearities [7]. In addition, slow SSMs can be used for model order reduction purposes, as they contain trajectories that remain active for the longest time before decaying to a NNM. We illustrate these ideas on several mechanical examples.

References

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