The threshold behaviour of chaotization phenomenon for multiple frequency perturbations in a cell

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Summary. We consider the chaotic motion of a particle in a cell on condition that the external action has a finite discrete spectrum and the particle interacts with its neighbors by means of a non-linear potential. The numerical experiments were carried out for Lennard-Jones potential. It was shown, that in the case of a single spectrum the chaotization onset is of a threshold type in both amplitude and frequency. The multi-frequency external excitation was used to form a global chaos. We showed that in this case the global chaotization starts at excitation amplitudes and frequencies far below their critical values for single-frequency excitation.

Introduction

One-dimensional chains of particles were extensively studied as chaotic dynamics problems [1] by reducing to the well-known standard maps. The study of chaos onset in the oscillations of a chain of particles is also important in relation with the application of the particle method to modelling material properties [2]. Although computer modelling can in many cases adequately describe the behaviour of materials, the problem of understanding this behaviour from the point of view of nonlinear dynamics remains.

For a one-dimensional system, the corresponding problem can be reduced to a study of motion of a particle in a cell. On the left side of the cell the particle interacts with a stationary particle and on the right side it interacts with a particle moving according to an external law $\xi(t)$. The inter-particle interaction is characterized by a non-linear potential. The asymptotic analysis is applicable to potentials of a general form, while the numerical results are presented for a Lennard –Jones potential.

Resonance overlap criterion

The dynamics of this particle is determined by a Hamiltonian and we use action-angle variables (I,φ) to analyse system's dynamics near the nonlinear resonance in the vicinity of an elliptic point. Consider invariant curves corresponding to resonance action values $I_{n1}, I_{(n-1)1}$. In order to generate a chaotic region it is necessary to destroy the invariant curves corresponding to these action values. The analysis shows [1] that the onset of chaotization coincides with the resonance overlap. The corresponding criterion proposed by B.V. Chirikov [3] states that the overlap of adjacent resonances leads to appearance of chaotic regions when the sum of resonances' half-widths ΔI is greater than the distance δI. Introducing parameter $K_{n,n-1} = \Delta I / \delta I$, characterizing the degree of resonance overlap [1], Chirikov's criterion can be written as $\left|K_{n,n-1}\right| \sim 1$.

Global chaos under a single frequency excitation

Let us consider the mechanism of the global chaos onset in the vicinity of an elliptic point under one-frequency excitation $\xi(t) = \alpha_1 \cos \omega_1 t$ where $|\alpha_1| << 1$. The effect of critical amplitude and frequency values in the formation of global chaos can be seen in Fig. 1., Fig. 2. They show the Poincare sections corresponding to several values of excitation amplitude one of them $\alpha_1 = 0.006$ is smaller than a critical value and another $\alpha_1 = 0.008$ is greater than a critical value.

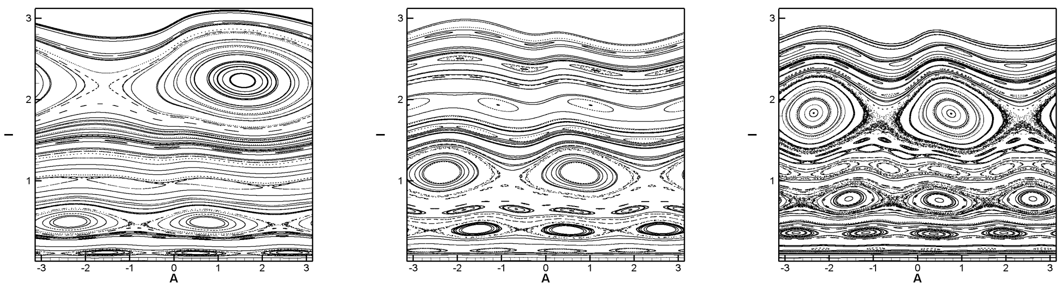

Fig. 1. Poincare sections for $\alpha_1 = 0.006$ in (I,φ) coordinates. Excitation frequencies are: $\omega_1 = 50$ in the left column, $\omega_1 = 70$ in the central column and $\omega_1 = 90$ in the right column.

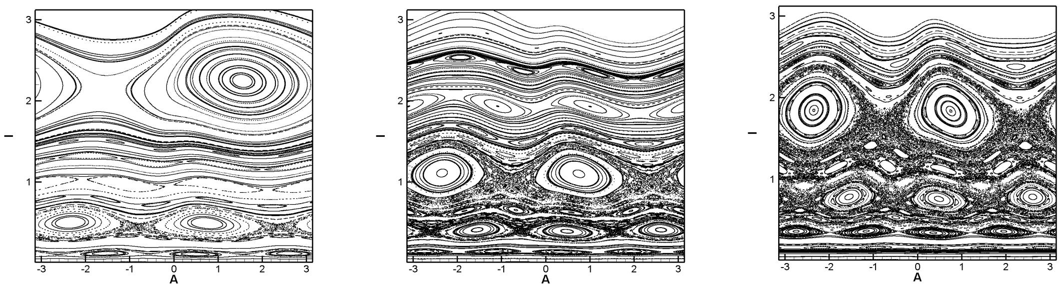

Fig. 2. Poincare sections for $\alpha_1 = 0.008$ in (I, φ) - coordinates. Excitation frequencies are: $\omega_1 = 50$ in the left column, $\omega_1 = 70$ in the central column and $\omega_1 = 90$ in the right column.

Multi-frequency excitation

Consider a motion of the system for multi-frequency excitation $\xi(t) = \sum_{i=1}^{N} \alpha_i \cos \omega_i t$. The Poincare sections for double-frequency, triple-frequency and quadruple-frequency cases are presented in Fig. 3, Fig. 4. The total amplitude was chosen to be smaller than the critical amplitude for a single-frequency excitation.

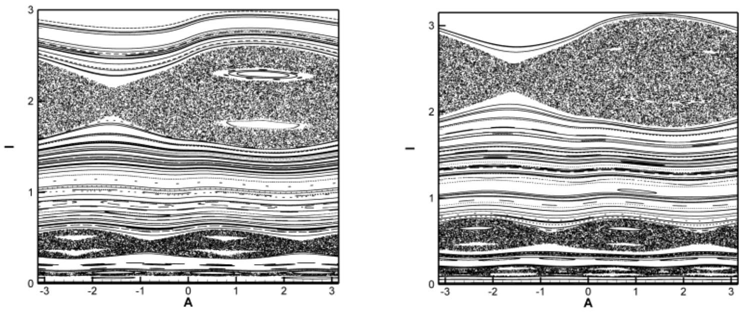

Fig 3. Poincare sections for double-frequency excitation. On the left: $\alpha_1 = \alpha_2 = 0.002$ and $\omega_1 = 45$, $\omega_2 = 55$; on the right: $\alpha_1 = \alpha_2 = 0.002$ and $\omega_1 = 50$, $\omega_2 = 55$.

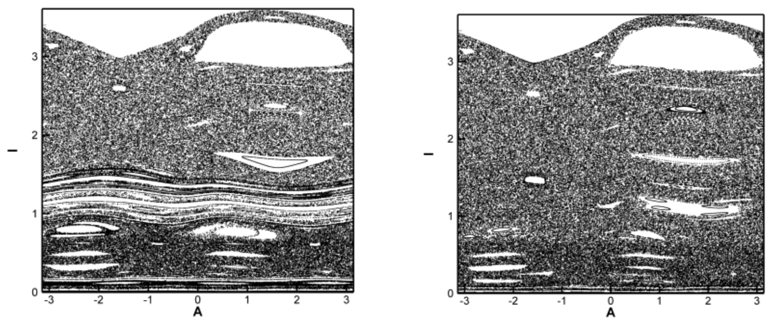

Fig. 4. Poincare sections for triple-frequency and quadruple-frequency excitations. On the left: $\alpha_1 = \alpha_2 = \alpha_3 = 0.002$ and $\omega_1 = 44$, $\omega_2 = 50$, $\omega_3 = 60$, on the right: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.002$ and $\omega_1 = 36$, $\omega_2 = 44$, $\omega_3 = 50$, $\omega_4 = 60$.
The presented results demonstrate that a multi-frequency excitation, even of a small amplitude, can lead to a substantial increase of system chaotization. Note that in this case there is practically no threshold value of the critical excitation amplitude and frequency.

References

[1] Zaslavsky GM. (2007) The Physics of Chaos in Hamiltonian Systems. Imperial College Press.
[2] Norman G.E., Stegailov V.V. (2001) The Stochastic Properties of a Molecular-Dynamical Lennard-Jones System in Equilibrium and Nonequilibrium States. *J. Experimental and Theoretical Ph.* **92**: 879–886.
[3] Chirikov BV. (1979) A universal instability of many-dimensional oscillator systems. *Phys Rep*; **52**:263–379.