Periodic Regimes Caused by Ice-Fluid-Simple Oscillator Interaction

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Summary. A new model describing the periodic vibrations caused by the interaction between the moving ice floe and a simple oscillator is proposed. The equations obtained estimate the forces affecting the oscillator under its interaction with ice. The influence of the velocity of ice movement on different vibration regimes is analyzed.

The problem of vibration of a structure caused by interaction with the ice floe has been studied for the past forty-five years, but has not yet been solved conclusively [1–3]. This paper considers a new model of contact interaction between the structure (oscillator) and the ice floe moving at a constant velocity. This model makes it possible to obtain an equation for the contact interaction force. In general, the contact zone is a mixture of water and ice pieces generated in the zone upon achieving critical pressure. The latter definitely depends on the ice rheology, which is not considered in this work and is regarded as known. When ice is moving with a constant velocity, the ice–water mixture has a variable composition due to formation of new ice pieces and washout of old ice. This process is accompanied by changes in the contact zone width. Hence, the formalization of the behavior of the environment between the oscillator and the moving ice body is reduced to description of a two-component continuous medium (water ice). This description takes into account both the formation of ice pieces and their withdrawal upon moving through the side gaps. Depending on the velocity, the ice floe pieces have time or do not have time to wash out from the space between the structure and the ice floe edge. These scenarios of the behavior of the environment lead to different structure oscillation conditions. If it is assumed for simplification purposes that the ice concentration in the water–ice environment is low relative to the water concentration, then the basic equations for the environment interacting with an oscillator will be as follows:

$$-\frac{\partial \tilde{p}}{\partial x} = \rho_0 \frac{\partial v_x}{\partial t} + Q_x (v_x - v_{xi}); \quad -\frac{\partial \tilde{p}}{\partial y} = Q(v_y - v_{yi}) + \rho_0 \frac{\partial v_y}{\partial t};$$

$$\frac{\partial \tilde{\rho}}{\partial t} + \rho_0 \frac{\partial v_x}{\partial x} + \rho_0 \frac{\partial v_y}{\partial y} = 0; \quad \tilde{p} = c_0^2 \tilde{\rho}; \quad -a \le y \le a; \tag{1}$$

$$\tilde{p} = c_0^2 \tilde{\rho}; \quad q(t) \le x \le s(t); \quad s = s_0 - v_0 t + \sum_{k=1}^{\infty} \Delta_k H(t - t_k).$$

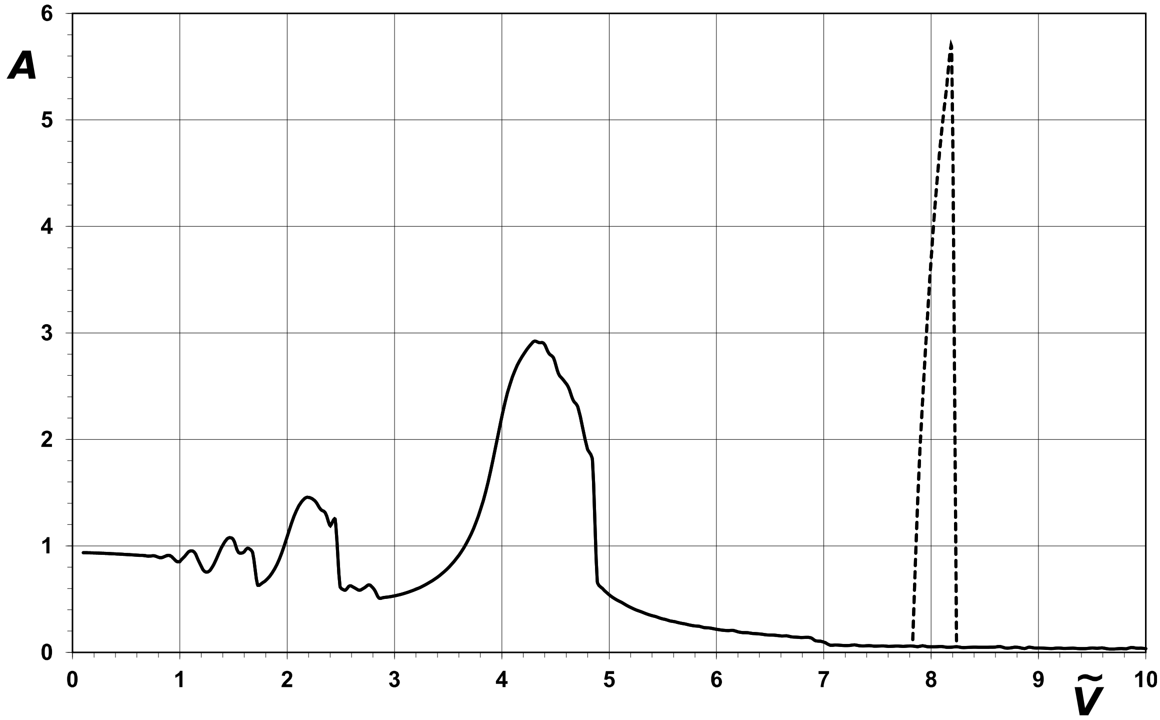
This system of equations assumes an acoustic approximation for water. The boundary conditions are taken as $\tilde{p}\big|_{y=\pm a} = 0$ and generally can be complicated with account for the interaction environment. In the system (1): $\tilde{p}, \tilde{\rho}$ are excessive pressure and water density, ρ_0 is the water density, v_x, v_y are components of the water velocity vector on the axes *x* and *y*, respectively, v_{xi}, v_{y_i} are components of the ice pieces velocity vector, $Q_x(v_x - v_{xi})$, $Q_y(v_y - v_{yi})$, are forces controlling the interaction between water and ice pieces under ice flowing round towards the axes *x* and *y*, c_0 is the velocity of sound in water, *t* is time, *q(t)* is the oscillator displacement, *s(t)* is the distance between the moving ice edge and the oscillator, *a* is the half-width of the oscillator body, V_0 is constant ice floe velocity, Δ_k is the size of ice pieces that have broken off, $H(t-t_k)$ is the Heaviside function, a t_k re the moments of time when ice is broken, which are related to the time when the water pressure reaches a value resulting in the breaking of the ice floe. It is assumed in Eqs. (1) that the major contribution to the environment pressure in the gap is made by liquid pressure in the direction *y*, rather than in the direction *x*; therefore, the force $Q_x(v_x - v_{xi})$ can be neglected. The force $Q_y(v_y - v_{yi})$ controls the basic resistance of the environment when ice pieces are pushed out in the lateral direction. In the first approximation, the velocity of ice pieces can be neglected relative to the water velocity in the contact zone and the equation for $Q_y(v_y - v_{yi})$ can be given as $Q_y = \beta v_y$.

The factor β depends on the number of broken ice pieces *N* occurring in the space between the travelling edge of the ice cover and the oscillator, and it is proportional to the value $\frac{k(N)}{s-q}\rho_0$. The inertia forces $\rho_0 \frac{\partial v_x}{\partial t}$ are neglected, and the first equation of system (1) is determined roughly, under the consideration that $\tilde{p}$ does not depend on the coordinate *x*. Using the shallow water model and averaging by the *x* coordinate of the third equation in system (1), the following system for the structure dynamics is obtained:

$$\dot{p} + \alpha(s - q)p = -\beta\frac{\dot{s} - \dot{q}}{s - q}; \quad M\ddot{q} + Gq = -Jp(t); \quad \alpha = \frac{10}{12a^2K(N)}\rho_0 c_0^2; \quad \beta = \frac{5}{12a^2}\rho_0 c_0^2;$$

$$s = s_0 - v_0 t + \sum_{k=1}^{\infty}\Delta_k H(t - t_k); \quad t_k = \frac{p^*}{1 + p^*}\cdot\frac{s_0 + \sum_{k=1}^{m}\Delta_k H(t - t_k)}{v_0}. \tag{2}$$

In system (2), the value p^* corresponds to the pressure value under which ice pieces are broken off and M and G are the oscillator mass and hardness and $\int_{-a}^{a}\tilde{p}dy = Jp(t)$. Note that for Δ_k constant the pressure $p(t)$ is a periodic in time function. Before the solution, system (2) was reduced to the dimensionless form. Figure 1 shows the dependence of the oscillator amplitude on stationary regimes on a floe dimensionless velocity $\tilde{V} = \frac{V_0\sqrt{GM}}{Jp^*}$. The values related to maximums correspond to frequencies that have the following characteristics: $\tilde{V}$ = 4.4, the system resonance at a frequency that is close to the oscillator frequency; $\tilde{V}$ = 8.4, at a frequency nearly corresponding to the system parametric resonance; $\tilde{V}$ = 2.1, at half of the oscillator frequency (subharmonics frequency).



Conclusions

The proposed model of the oscillator behavior under the action of fluid flow mixed with ice adequately describes two out of three possible oscillation regimes caused by the interaction of the structure with the moving ice field: intermittent and synchronization.

References

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