Control of mechanical systems with uncertain set-valued friction†

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Summary. We present a control architecture for the set-point stabilization of motion systems subject to set-valued friction, including a velocity-weakening (Stribeck) effect. The proposed controller consists of a switching PID term and a term that robustly compensates for the Stribeck effect. It is shown that the controller asymptotically stabilizes the set-point, and that this controller is robust for unknown static friction, and an uncertain contribution of the Stribeck effect.

Introduction

It is well known that friction is a performance limiting factor in high-precision positioning systems. Many different approaches have been presented in literature that deal with the effects of friction in order to achieve high precision positioning. For instance, friction compensation techniques use a friction model in the controller, but modeling errors are inevitable such that positioning accuracy is compromised due to over- or undercompensation of the friction, see e.g., [1, 2]. Non-compensation-based techniques, such as PID control for systems subject to friction with the Stribeck effect may lead to limit cycling, whereas impulsive control may lead to the excitation of high-frequency system dynamics. In this work, we present a control technique that guarantees set-point stabilization of a motion system subject to static friction and the velocity-dependent Stribeck effect. The proposed control architecture will be shown to be robust for unknown static friction and an uncertain Stribeck effect, and does not excite high-frequency system dynamics.

Controller design

Consider a controlled sliding inertia m subject to a friction characteristic that consists of unknown static friction (F_s) and an uncertain velocity-dependent friction contribution $f(v)$, as visualized in Figure 1. The system dynamics are described by the following differential inclusion:

$$\begin{aligned} \dot{x}(t) &= v(t), \\ \dot{v}(t) &\in \left(-F_s\text{Sign}(v) + f(v) - u(t)\right)/m, \end{aligned} \tag{1}$$

where x is the position, v the velocity and Sign represents the set-valued sign function, that is $\text{Sign}(y) = \text{sign}(y)$ for $y \neq 0$, and $\text{Sign}(y) \in [-1,\ 1]$ for $y = 0$. The smooth part of the friction $f(v)$ is considered to be potentially uncertain. The only assumption adopted for $f(v)$ is that it satisfies the inequality

$$vf(v) \leq -|v|\tilde{F}\left(e^{-\delta|v|} - 1\right), \tag{2}$$

for all $v \in \mathbb{R}$, as visualized by the dashed line in the right subplot of Figure 1. Herein, $\tilde{F} \in \mathbb{R}_{>0}$ is chosen such that the magnitude of the velocity-dependent friction part is bounded by $\tilde{F}$. The parameter $\delta \in \mathbb{R}_{>0}$ embeds a velocity-dependency in this bound, while still allowing to characterize the Stribeck (velocity-weakening) effect commonly present in frictional contact. The controller u consists of a switched PID term, and a term that robustly compensates for velocity-dependent and uncertain frictional effects (including the Stribeck effect), using the bound (2), such that the set-point $x = 0$ is stabilized. Integrator action plays an important role, as it is required to compensate for the uncertain static friction such that the system can escape the stick phase. The switching dynamic control law is given by

$$\dot{\xi} = \begin{cases} |x|, & \text{if } t < t_1 \vee \left(t \geq t_1 \wedge \xi < \bar{\xi}\right), \\ 0, & \text{if } t \geq t_1 \wedge \xi = \bar{\xi}, \end{cases} \tag{3a}$$

$$u = \underbrace{k_p x + k_d v - k_i\xi\left(\text{sign}(v) - (1 - |\text{sign}(v)|)\,\text{sign}(x)\right)}_{\text{switching PID term}} \underbrace{-\tilde{F}\left(e^{-\delta|v|} - 1\right)\text{sign}(v)}_{\text{vel. dependent compensation}}, \tag{3b}$$

where

$$\bar{\xi} = \xi(t_1) + \left(k_p|x(t_1)|\right)/k_i, \tag{3c}$$

$$t_1 := \max\left\{t \in \mathbb{R}_{\geq 0} \mid v(s) = 0 \wedge \dot{x}_2(s) = 0,\ \forall s \in [0, t]\right\} \tag{3d}$$

with initial conditions $|x(0)| \leq F_s/k_i$, $v(0) = 0$ (i.e., the system is in the stick phase, see [2]), and $\xi(0) = 0$. The proportional, derivative, and integral control gains of the PID term in (3b) are denoted by k_p, k_d, $k_i \in \mathbb{R}_{>0}$, respectively The integrator dynamics (3a) results in a *saturated* integrator state $\xi \in \mathbb{R}_{\geq 0}$, which cannot exceed a maximum value defined by $\bar{\xi}$. The purpose of this bound is that the integral control force can only compensate for no more than the

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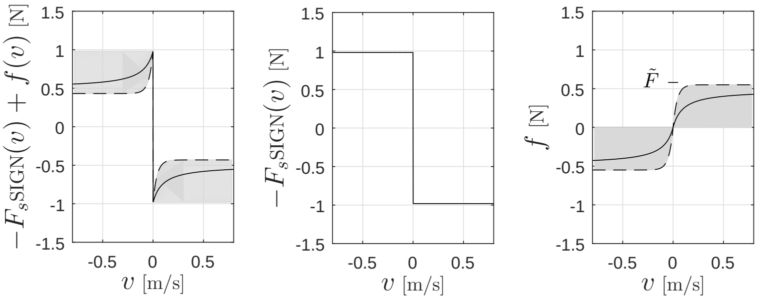

Figure 1: The friction characteristic (left) can be decomposed into static friction (center) and a velocity-dependent curve (right). The dashed line indicates the bound used in the controller.

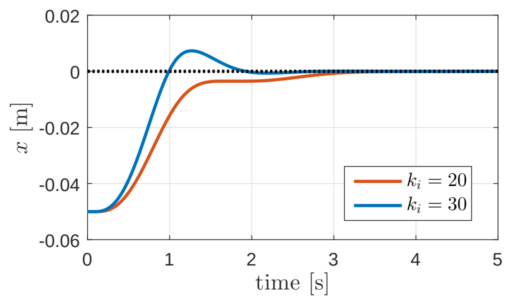

Figure 2: Simulated response of the closed-loop system for two values of the integrator gain k_i.

static friction. Its value is determined at the first stick-slip transition of the system, at $t = t_1$, see (3d). Moreover, in this particular design of the integrator the *absolute value* of the position error is integrated over time. The switching term $\mathrm{sign}(v) - (1 - |\mathrm{sign}(v)|)\,\mathrm{sign}(x)$ in (3b) is then employed to determine the sign of the integrator control force. If $v \neq 0$, the sign of the velocity v determines the sign of the integrator control force, whereas the sign of the position error x determines the sign of the force in case $v = 0$. In this way, the contribution of integrator action is still present when the system is in the stick phase ($v = 0$). If overshooting the set-point occurs, a conventional integrator has to change sign by gradually depleting and refilling its buffer in order to provide integral action in the correct direction. This process is slow, and leads to large settling times. The proposed controller, instead, leads to considerably faster convergence when overshoot of the set-point occurs, by using switching controls. Also note that the controller robustly stabilizes the set-point for any velocity-dependent friction contributions that lie inside the grey area in Figure 1.
Closed-loop stability of the system is analyzed as follows. First, it is shown that the (saturated) integrator state ξ, satisfies $\xi \leq \bar{\xi} = F_s/k_i$. This fact is then used to show that the set $\{(x, v, \xi) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \subset \mathbb{R}^3 \mid x = v = 0,\ 0 \leq \xi \leq F_s/k_i\}$ is asymptotically stable, using a Lyapunov stability analysis and a LaSalle-type invariance argument.

Illustrative example

Consider the closed-loop system (1), (3), with parameter values $m = 1$ kg, $k_p = 18$, $k_d = 1$, and either $k_i = 20$ or $k_i = 30$. The *true* friction characteristic is given by the solid curve visualized in Figure 1, with a pronounced Stribeck effect. The bound on the Stribeck curve, with $\delta = 25$ and $\tilde{F} = 0.55$ N, satisfies in the particular bound as shown by the dashed line in Figure 1. A numerical time-stepping routine is employed to numerically compute solutions of the closed-loop system, with initial conditions $x(0) = -0.05$, $v(0) = \xi(0) = 0$. The response of the closed-loop system is visualized in Figure 2. For $k_i = 20$, the response converges to the set-point without overshoot, whereas for $k_i = 30$, the response overshoots the set-point. In contrast to approaching the set-point from one side, overshooting the set-point results in velocity-reversals, and consequently in discontinuities in the control force, see Figure 3. These discontinuities occur at velocity reversals only, and, due to the particular design of the integrator, the integrator control force does not exceed the value of the static friction. Therefore, the integrator control force and static friction cancel each other such that the remaining dynamics consist of merely a PD-controlled inertia. Hence, the discontinuity in the control force does not result in motion of the system and will therefore not excite high-frequency system dynamics.

Conclusion

A controller for the set-point stabilization of motion systems subject to set-valued Stribeck friction is proposed. The controller renders the set-point asymptotically stable, and it is robust for unknown static friction and an uncertain Stribeck curve.

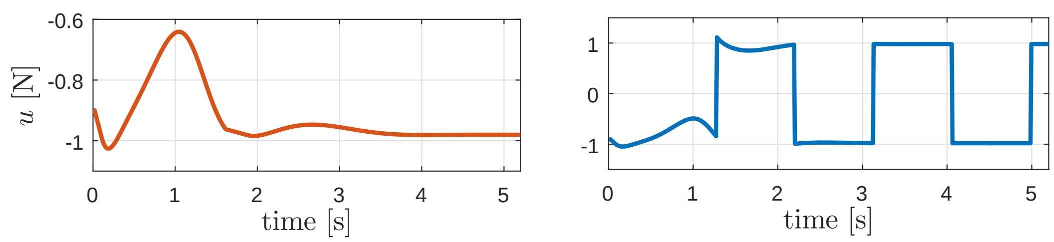

Figure 3: Control force u for a simulation with $k_i = 20$ (left) and $k_i = 30$ (right).

References

[1] Armstrong-Hélouvry, B., Dupont, P., Canudas de Wit, C., "A survey of models, analysis tools and compensation methods for the control of machines with friction", *Automatica*, vol. 30, no. 7, pp. 1083-1138, 1994.

[2] Putra, D., Nijmeijer, H., van de Wouw, N., "Analysis of undercompensation and overcompensation of friction in 1DOF mechanical systems", *Automatica*, vol. 43, no. 8, pp. 1387-1394, 2007.