String stability for cascaded systems subject to disturbances

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Summary. A notion of string stability for large-scale cascaded (nonlinear) systems is presented that provides a characterization of the amplification of disturbances as they propagate through the cascade. This notion is particularly relevant for the stability and performance analysis of groups of closely-spaced automatically controlled vehicles known as platoons and extends earlier string stability notions in the literature. The results are used to design a distributed controller for a platoon of vehicles.

Introduction

In large-scale cascaded (nonlinear) systems, the concept of string stability provides a notion of stability and performance by characterizing the attenuation of disturbances as they propagate through the subsystems in the cascade. This notion is particularly relevant for the performance analysis of groups of closely-spaced and automatically controlled (heavy-duty) vehices, see, e.g., [3, 2]. However, existing notions of string stability do not allow for the practically relevant case of external disturbances on vehicles. Motivated by this observation, we propose a notion of string stability for cascaded systems that includes disturbances on each subsystem. The results are illustrated by designing a distributed controller that achieves this notion of disturbance string stability, see also [1].

String stability

Consider the cascaded system of length $N+1$ as

$$\begin{aligned} \dot{x}_0 &= f(x_0, 0, w_0), \\ \dot{x}_i &= f(x_i, x_{i-1}, w_i), \quad i \in \mathcal{I}_N \backslash \{0\} \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$ for $i \in \mathcal{I}_N = \{0, 1, \ldots, N\}$ and f satisfying $f(0,0) = 0$ is locally Lipschitz. In (1), $w_i \in \mathbb{R}^m$, $i \in \mathcal{I}_N$ represent external disturbances on each of the subsystems. For cascaded systems of the form (1), the stability notion of disturbance string stability is defined as follows.

Definition 1. *The cascaded system (1) is said to be disturbance string stable if there exist functions $\bar{\beta}$ of class $\mathcal{KL}$ and $\bar{\sigma}$ of class $\mathcal{K}_\infty$ such that, for any initial condition $x_i(0)$ and bounded disturbance w_i, $i \in \mathcal{I}_N$, the solution x_i, $i \in \mathcal{I}_N$, exists for all $t \geq 0$ and satisfies*

$$\max_{i \in \mathcal{I}_N} |x_i(t)| \leq \bar{\beta}\left(\max_{i \in \mathcal{I}_N} |x_i(0)|, t\right) + \bar{\sigma}\left(\max_{i \in \mathcal{I}_N} \|w_i\|_\infty^{[0,t]}\right), \tag{2}$$

for all $N \in \mathbb{N}$. Here, $|x| = \sqrt{x^{\mathrm{T}} x}$ and $\|w\|_\infty^{[0,t]} = \sup_{\tau \in [0,t]} |w(\tau)|$.

In Definition 1, the requirement that (2) holds for all $N \in \mathbb{N}$ is crucial. Namely, this condition implies that perturbations cannot grow unbounded as they propagate through the subsystems in the cascade, even in case the cascade length grows unbounded. In addition, this condition ensures scalability as subsystems can be added to (or removed from) the cascade without affecting disturbance string stability properties. Finally, it is remarked that this notion extends existing notions of string stability [3] by explicitly accounting for the effect of external disturbances.
As the definition of disturbance string stability is based on properties on the entire cascaded system (even for all $N \in \mathbb{N}$), it could be difficult to evaluate in practice. The following result allows for establishing disturbance string stability on the basis of *local* properties, i.e., on the basis of properties of the individual subsystems.

Theorem 1. *Consider the cascaded system (1) and let each subsystem $i \in \mathcal{I}_N$ be input-to-state stable with respect to x_{i-1} and w_i, i.e., there exist functions β of class $\mathcal{KL}$ and γ and σ of class $\mathcal{K}_\infty$ such that*

$$|x_i(t)| \leq \beta\big(|x_i(0)|, t\big) + \gamma\big(\|x_{i-1}\|_\infty\big) + \sigma\left(\|w_i\|_\infty^{[0,t]}\right), \tag{3}$$

for all $i \in \mathcal{I}_N$, $N \in \mathbb{N}$ (and with $x_{i-1} = 0$ for $i = 0$). If the gain function γ satisfies

$$\gamma(r) \leq \bar{\gamma} r \tag{4}$$

for some $\bar{\gamma} < 1$, then the cascaded system (1) is disturbance string stable as in Definition 1.

The condition (4) specifies that each subsystem should strictly attenuate any perturbations that are the result of its predecessor (through the coupling with its state x_{i-1}).

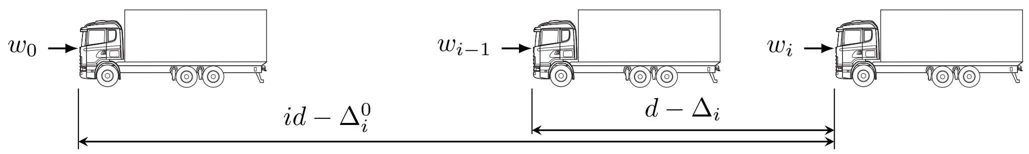

Figure 1: A platoon of heavy-duty vehicles and the spacing errors Δ_i and Δ_i^0 of vehicle i with respect to its predecessor and the lead vehicle, respectively.

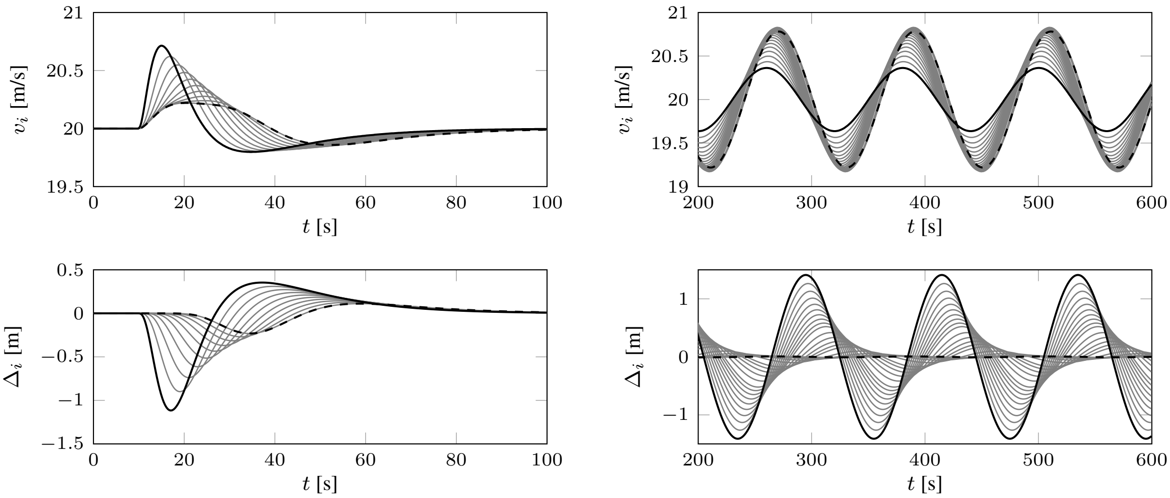

Figure 2: Velocities v_i (top) and spacing errors Δ_i (bottom) for platoons subject to external disturbances. In the left figures, a platoon for $N = 10$ is considered in which only the lead vehicle encounters a disturbance. In the right figures, a platoon for $N = 50$ is considered and all vehicles are subject to disturbances. The parameter values of the relaxed spacing policy (5) are $\kappa = 2$ and $\kappa_0 = 0.1$.

Example: Control design for string stability of vehicle platoons

A platoon of $N + 1$ vehicles is considered as in Figure 1, where each vehicle $i \in \mathcal{I}_N$ has a position s_i, velocity v_i, and potentially more states describing, e.g., acceleration or engine dynamics. Each vehicle is subject to an external disturbance w_i representing, e.g., the effect of road slope or wind. For such a platoon, the objective is to design a distributed controller that achieves tracking of a constant inter-vehicular distance as well as disturbance string stability. Here, the tracking errors $\Delta_i = s_i - s_{i-1} + d$ and $\Delta_i^0 = s_i - s_0 + id$ denote the spacing error for vehicle i with respect to its predecessor and the lead vehicle (with index 0). However, rather than directly targeting these spacing errors, the relaxed spacing policy

$$\delta_i = (1 - \kappa_0)\Delta_i + \kappa_0 \Delta_i^0 + \kappa(v - v_{\text{ref}}) \tag{5}$$

is defined, where $0 \leq \kappa_0 < 1$ and $\kappa > 0$. Namely, the following result holds.

Theorem 2. *For any controller that achieves, for some functions β of class $\mathcal{KL}$ and σ of class $\mathcal{K}_\infty$,*

$$|\delta_i(t)| \leq \beta\big(|\delta_i(0)|, t\big) + \sigma\big(\|\bar{w}_i\|_\infty^{[0,t]}\big), \tag{6}$$

a platoon with relaxed spacing policy (5) is disturbance string stable if $\kappa_0 > 0$.

Consequently, the inclusion of lead vehicle information in platoon control leads to string stable behavior. Here, it is noted that the results of Theorem 2 are independent of the details of the vehicle model, as long as the (potentially nonlinear) vehicle model is feedback linearizable. The performance of a platoon subject to a distributed controller satisfying (6) is evaluated in Figure 2.

Conclusions

In this abstract, a notion of string stability for cascaded systems subject to external disturbances is presented. This stability notion is scalable and sufficient conditions are given in terms of properties of individual subsystems. Finally, the usefulness of this notion is illustrated by application to a platoon of heavy-duty vehicles.

References

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