ENOC 2017, June 25-30, 2017, Budapest, Hungary

Experimental Validation of Vibro-Impact Force Models using Numeric Simulation and Perturbation Methods

Geraldo F. de S. Rebouças[†], Ilmar F. Santos and Jon J. Thomsen
Department of Mechanical Engineering, Technical University of Denmark – DTU, Denmark

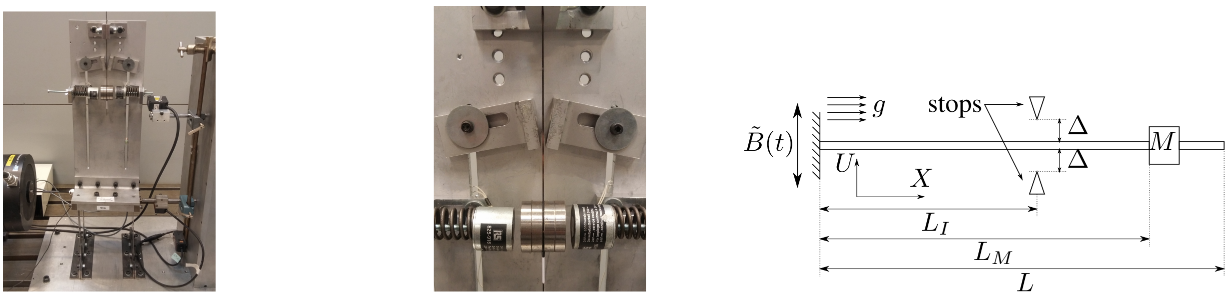
Summary. The frequency response of a single-degree of freedom vibro-impact oscillator is analysed using Harmonic Linearization, Averaging and Numeric Simulations considering two different impact force models, one given by a piecewise-linear function and other by a high-order polynomial. Experimental validation is carried out using control-based continuation to obtain the experimental frequency response, including its unstable branch.

Introduction

The objective and main originality of this work is to compare different impact force models using analytical, numeric and experimental techniques. The frequency response of a single-degree of freedom vibro-impact oscillator is analysed using Harmonic Linearization and Averaging considering two different impact force models, one given by a piecewise-linear function and other by a polynomial one, see [4]. Experimentally, control-based continuation [2, 3] is used to obtain the frequency response of an impacting beam, including its unstable branch. Numeric simulations are used to validate simple analytic approximations obtained by perturbation methods.

Experimental Setup

The experimental setup has been described previously by [2, 3], where a model-free controller was tuned for performing control-based continuation, obtaining the system's frequency response and analysing the stability of the orbits found. It consists of a cantilever beam with lumped mass attached to a platform, which is connected to an electrodynamic shaker, see Fig. 1a. A pair of symmetrically situated stops restrain the lateral movement of the beam, and two electromagnetic actuators are placed on each side of the mass to execute control-based continuation.



(a) Test rig with shaker, actuators and sensors.

(b) Detailed view of the beam.

(c) Mechanical Model.

Figure 1: Experimental setup and mechanical model.

The lumped mass dynamics dominates the oscillations in comparison to the flexible beam alone. The lumped mass is located with a reasonable distance from the stops with clearance, which can be considered as rigid supports, causing near-elastic impacts.

Mathematical Modeling

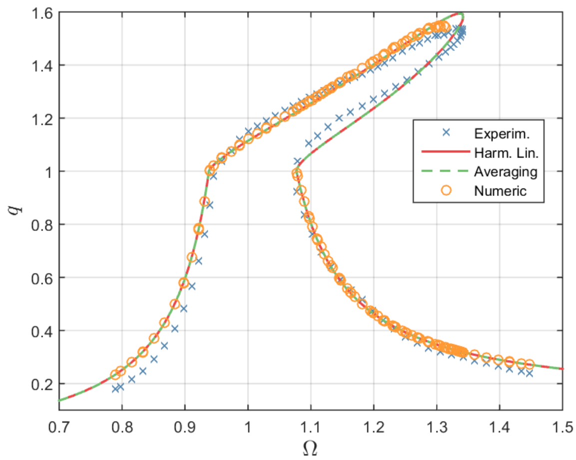
The Galerkin method is used to discretize a Bernoulli-Euler model for the test beam, giving a single DOF nondimensional model: $\ddot{q} + 2\beta_s\dot{q} + q + F_I(q, \dot{q}) = \Omega^2 b \sin(\Omega t)$, where q is a modal displacement coordinate, β_s is the structural damping coefficient, b and Ω are the forcing amplitude and frequency. The amplitudes of modal oscillations and forcing excitations are normalized by the gap width, $\Delta \approx 1.6\,\mathrm{mm}$, and the forcing frequency is normalized by the system's fundamental linear natural frequency, $f_n \approx 7.6\,\mathrm{Hz}$. In order to apply harmonic linearization and averaging, one can use the following assumptions: 1) damping, forcing and nonlinear terms are weak; and 2) a mono-frequency first order solution, i.e., $q(t) = Q\sin(\Omega t + \varphi)$. The impact force $F_I(q, \dot{q})$ can be approximated by a polynomial, according to [4] or as a piecewise linear function, [1]. The impact force is a combination of restoring and dissipative forces. It can be approximated by a polynomial in the modal displacement, $F_I(q, \dot{q}) = f_e q^{2n-1} + 2\beta_I q^{2p}\dot{q}$, where f_e is the impacting restoring force coefficient and β_I is the impacting damping. The piecewise linear impact force appears only when the displacement exceeds the gap, i.e. $|q| \geq 1$. If $q \geq 1$ the force is equal to $(\omega_I^2 - 1)(q - 1) + 2\beta_I\dot{q}$ and $(\omega_I^2 - 1)(q + 1) + 2\beta_I\dot{q}$ if $q \leq -1$, otherwise it is zero. Here, ω_I is the non-dimensional impacting linear natural frequency. In terms of model

[†] gerreb@mek.dtu.dk

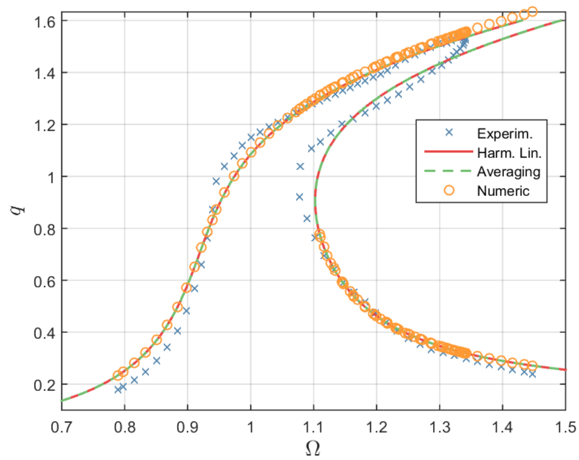
tuning the piecewise-linear model contains fewer parameters than the polynomial one. Numerically, the discontinuity of the piecewise linear model makes its simulation more demanding. The same can be said for polynomial force with high values of n and p.

Results

The aforementioned perturbation methods can be used to obtain the approximate frequency response of the impact oscillator as an implicit frequency-amplitude relationship. To obtain the numeric values of the model parameters, this relationships are fitted to the experimental data obtained by control-based continuation. The result is $\beta_I = 60 \times 10^{-3}$, $f_e = 0.12$, $n = 4$ and $p = 1$ for polynomial force model; $\omega_I = 4$ and $\beta_I = 0.16$ for piecewise linear force; and $\beta_S = 18 \times 10^{-3}$, $b = -0.14$ for structural damping and forcing amplitude. To check the validity of the assumptions made, the fitted values are used to obtain the frequency response of the system by numeric simulation. The results are shown in Fig. 2, where one can see that both force models give results reasonably close to experimental observations. Also, one can see that the perturbation methods are equivalent, giving the same frequency response. Thus, the assumptions made appears adequate. Comparing the force models, it is easy to see that the piecewise-linear model is able to predict the experimental behaviour more precisely, capturing both fold points, while the polynomial model fails to predict the upper folding point and has its lower fold point further away from experimental data. The precision of the piecewise linear model agrees with results found on literature, such as [5], where averaging was used to obtain the frequency response of a piecewise-linear isolator around resonance.



(a) Piecewise-linear impact force.



(b) Polynomial impact force.

Figure 2: Comparison of frequency responses.

Conclusions

Two different impact force models (polynomial and piecewise-linear) were analysed and validated against experimental observations. Averaging and Harmonic Linearization were used to obtain the system's frequency response and to tune the force models using experimental data. The results show that both perturbation methods provide equivalent results for the system under analysis and that the frequency-amplitude relationship obtained can be used to obtain numeric values for model parameters. Regarding the impact force models, the piecewise-linear force seems to describe the frequency response more accurately, predicting both folding points while the polynomial force predicts only one and not so accurately as the piecewise linear force.

Further experimental validation of other impact models, like kinematic impact with coefficient of restitution and analysis by nonsmooth transformations [6], can be relevant to compare different vibro-impact models in terms of ease of use, applicability and reliability.

References

[1] Babitsky, V. I. (1998) Theory of Vibro-Impact Systems and Applications. Springer, Berlin.

[2] Bureau E., Schilder F., Santos I. F., Thomsen J. J., Starke J. (2013) Experimental bifurcation analysis of an impact oscillator – Tuning a non-invasive control scheme. *J. of Sound and Vibration* **332**(22):5883-5897.

[3] Bureau E., Schilder F., Elmegård M., Santos I. F., Thomsen, J. J., Starke J. (2014) Experimental bifurcation analysis of an impact oscillator – Determining stability. *J. of Sound and Vibration* **333**(21):5464-5474.

[4] Hunt K. H., Crossley F. R. E. (1975) Coefficient of Restitution Interpreted as Damping in Vibroimpact. *J. of Applied Mechanics* **42**(2):440.

[5] Narimani A., Golnaraghi M. F., Jazar R. (2004) Frequency Response of a Piecewise Linear Vibration Isolator. *J. of Vibration and Control* **10**(12):1775-1794.

[6] Pilipchuk V. N. (2015) Closed-form solutions for oscillators with inelastic impacts. *J. of Sound and Vibration* **359**:154-167.