Evaluating Nonlinear Responses of Asphalt Concrete Mixtures under Time-Dependent Loading: in view of three representation functions

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Summary: This paper aims to evaluate nonlinear responses of asphalt concrete mixtures under time-dependent loading using three representation functions (Power Law, Prony series function, and Burger Model Based Representation function) and compares their differences in performance analysis. The paper presents the determination of nonlinear time-dependent mechanical behaviors of asphalt concrete involving a series of analyses including time-temperature superposition principle, pre-smoothing process, and least squared error trial, and nonlinear regression in order to generate parameters of representation functions for each one of representation functions. The objective of the paper is to assess what makes differences among the three functions by means of analyzing nonlinear responses of asphalt concrete mixtures when subjected to time-dependent loading. The analysis results conclude the Prony series and the BMBR function show good agreement with the BBR test results and both have better accuracy in predicting nonlinear responses of asphalt mixtures.

Introduction

This paper aims to evaluate nonlinear responses of asphalt concrete mixtures under time-dependent loading using three representation functions (Power Law, Prony series function, and Burger Model Based Representation function) and compares their differences in performance analysis. Hot mix asphalt concrete has been treated as a composite material consisting of asphalt binders, aggregates, and air voids from which its mechanical performance tends to be nonlinear and its responses is much more complicated for analysis and prediction. When subjected to time-dependent loading (weather variations, dynamic loads, etc.), asphalt concrete exhibits elastic behavior followed by a slow and continuous increase of strain at a decreasing rate known as a viscoelastic response [1]-[2]. In the past decades, relaxation modulus is used by most researchers to model such nonlinear mechanical behaviors of asphalt concrete mixtures [3]-[8]. However, direct measurement of relaxation modulus of asphalt mixtures requires robust instruments, which is not a favorable option in most laboratories [9]. An alternative measurement of creep compliance therefore becomes a commonly used method to evaluate the viscoelastic performance of asphalt concrete mixtures. Creep compliance data can be obtained from indirect tensile tests (IDT), bending beam rheometer (BBR), etc. To predict the viscoelastic responses, a mathematical representation model must be developed to form a viscoelastic material model (constitutive equation) that contains a time-dependent function of strain or stress along with an elastic domain and a viscous domain.

Approach

Several viscoelastic response functions have been used to characterize nonlinear dynamic behaviors of asphalt concrete mixtures. Power law and Prony series are two typical representation functions of creep compliance. More recently, a Burger Model Based Representation (BMBR) function developed by Ho at Northern Arizona University and Romero at the University of Utah has been proposed and available for researchers to be used in prediction of nonlinear behavior of asphalt concrete mixtures [10]. Ho and Romero indicated that the BMBR function can significantly reduce the complexity of parameter generations and does not involve with the sophisticated processes of Laplace transform and Laplace transform-inversion, thus making the viscoelastic analysis somewhat less complicated. Since the BMBR function is not as complicated as other representation functions, it would be ideal to include this function in prediction and analysis of nonlinear responses of asphalt concrete mixtures. The paper presents the determination of nonlinear time-dependent mechanical behaviors of asphalt concrete involving a series of analyses including time-temperature superposition principle, pre-smoothing process, and least squared error trial, and nonlinear regression in order to generate parameters of representation functions for each one of representation functions. The three representation functions (Power law, Generalized Maxwell function, and BMBR) were applied in prediction of nonlinear responses of asphalt concrete mixtures for their viscoelastic behavior/relaxation modulus. The objective of the paper is to assess what makes differences among the three functions by means of analyzing nonlinear responses of asphalt concrete mixtures when subjected to time-dependent loading.

Material Preparation and Testing

An asphalt paving project located in Flagstaff, Arizona, USA was used to material sampling. Asphalt mixtures collected from the job site were shipped to the Materials Laboratory of Northern Arizona University where all mixtures were reheated and compacted to specimens with a 150 mm in diameter and 110 mm in height using a Superpave gyratory compactor (SGC). All SGC specimens were further trimmed to rectangular specimen with a dimension of 4.5” x 4.5” x 2.5” (11.43 cm x 11.43 cm x 6.35 cm). All specimens were placed in an American Society for Testing and Materials (ASTM) apparatus (Figure 1) and were undertaken a series of freeze-thaw (F-T) tests at 0, 100, 150, 200, 250, and 300 cycles. After a desired F-T cycle test was completed, specimens were removed from the apparatus and experienced a time-dependent loading test using a bending beam rheometer (BBR) (Figure 2) to collect creep compliance data. The entire F-T cycle testing process including the dimensions of specimens are explained in the reference by Ho et al. [11] so the F-T testing procedure is neglected in the paper.

Figure 1: F-T cycle testing set up using an ASTM C666 apparatus (left) and specimens after a number of F-T cycles (right)

Figure 2: Close out of specimens (left) and bending beam rheometer (right)

Representation Functions of F-T cycled Asphalt Mixtures: Power Law Function

Hot mix asphalt (HMA) concrete consists of asphalt binders, aggregates, and air voids that form together as a composite material. At low temperatures, the HMA pavement exhibits a linear viscoelastic (LVE) performance in which representation functions must be used to simulate its mechanical behaviors. As of today, a number of numerical models have been used by researchers to predict LVE responses of asphalt mixtures. During the computational processes, the primary difficulty of LVE analysis is the processes of Laplace transform and Laplace transform-inversion converting in between creep compliance and relaxation modulus of the asphalt mixtures. As currently used, a generalized power law function and a prony series function have been used substantially as representation functions to predict relaxation modulus of the asphalt mixtures. Generalized power law function can be expressed as:

$$D_{GPL}(t) = D_0 + D_1 \cdot t^n \tag{1}$$

where

D_0 represents the value of elastic creep compliance,
D_1 refers to the value of creep compliance curve at time t,
n denotes the power parameter

Representation Functions of F-T cycled Asphalt Mixtures: Prony Series Function

In addition to power law functions, Prony series has also been widely used to be an analytical representation function in viscoelastic modeling. Prony series consists of one Maxwell model (a spring and a dashpot connected in series) and several Kelvin elements (a spring and a dashpot connected in parallel) as shown in Figure 3.

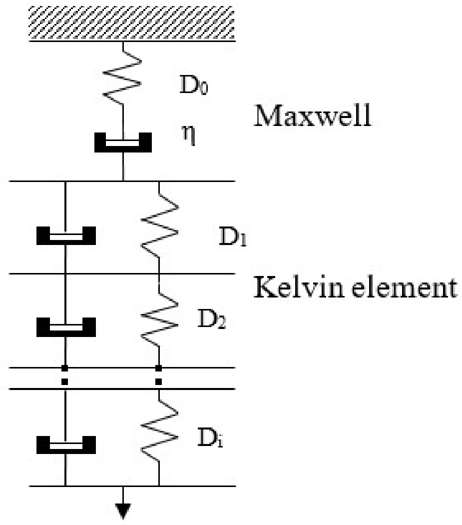

Figure 3: Generalized Maxwell model

The Prony series function can be mathematically expressed as:

$$D(t) = D_0 + \frac{1}{\eta}t + \sum_{i=1}^{N} D_i \left(1 - e^{-t/\tau_i}\right) \tag{2}$$

where D_0, D_i, and η= Prony series parameters; and τ_i =retardation times.

To derive the relaxation modulus of the asphalt mixtures from (1) and (2) require both Laplace transform and Laplace transform-inversion processes. The entire derivations have been presented by numerous papers Christenson [12] as shown below:

$$E_{GPL}(t)=\frac{1}{D_0+D_1\Gamma(n+1)(1.786t)^n} \qquad (3)$$

where: Γ is a gamma function

$$E_{\mathrm{Pr}ony}(t)=C_1e^{-\gamma_1}+C_2e^{-\gamma_2}+C_3e^{-\gamma_3}+\ldots+C_je^{-\gamma_j}=\sum_{j=1}^{N}C_je^{-\gamma_j} \qquad (4)$$

where

C_j= Prony regression coefficient,

$\gamma_j = \frac{t}{\lambda_j}$ in the viscoelastic analysis,

t = creep compliance times, and

τ_j= regression coefficient

Burger Model Based Representation Function

In 2012, a research by Ho and Romero [10] provided an alternative function to directly invert the Laplace transform of the Prony series function. Their function is structured with a Maxwell model connecting with a Kelvin element known as a Burger model (Figure 4). The Burger model based representation (BMBR) function was derived with the goal to significantly reduce the complexity of parameter generations and does not involve with the sophisticated processes of Laplace transform and Laplace transform-inversion. The advantage of the BMBR function is its simplicity in the LVE analysis as compared with the Prony series function.

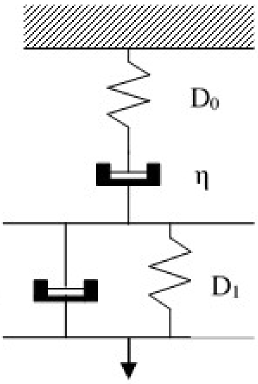

Figure 4: Components of a Burger model

The relaxation modulus of asphalt mixtures using the (BMBR) function is expressed from Eq. 5 through Eq. 12 [10]:

$$E(t)=C_1\times\left[e^{(\alpha-\beta)t}+e^{-(\alpha+\beta)t}\right]+C_2\times\left[e^{(\alpha-\beta)t}-e^{-(\alpha+\beta)t}\right] \qquad (5)$$

where:

$$\alpha=\sqrt{\frac{B^2}{4A^2}-\frac{C}{A}} \qquad (6)$$

$$\beta=\frac{B}{2A} \qquad (7)$$

$$A=D_0x\eta \qquad (8)$$

$$B= \quad D_0 \times \frac{\eta}{\tau_1} + D_1 \times \frac{\eta}{\tau_1} + 1 \tag{9}$$

$$C= \frac{1}{\tau_1} \tag{10}$$

$$C_1 = \frac{\eta}{2A} \tag{11}$$

$$C_2 = \frac{\frac{\eta}{\tau_1} - \eta \times \beta}{2A \times \alpha} \tag{12}$$

LVE Results of Representation Functions

This section evaluates the effectiveness of representation functions in fitting raw creep compliance data obtained from BBR tests at selected F-T cycles. Three representation functions were used: Prony series function with one Maxwell model plus six Kelvin element, Burger model (one Maxwell model plus one Kelvin element), and generalized Power law function. The analyses were performed at 0, 100, 150, 200, 250, and 300 F-T cycles. To evaluate the effectiveness of individual representation function fitting raw data, the root mean squared error (RMSE) was performed. This statistic measures the total deviation of the response values (creep compliance) between the raw data and individual creep compliance generated by representation functions (generalized Power law function, Prony series function, or Burger model). A value closer to 0 indicates that a fit representation function is more useful for prediction. The RMSE is an estimate of the standard deviation of the random component in the data, and is defined as:

$$RMS \quad = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x}_i)^2}{n}} \tag{13}$$

where:

x_i is the creep compliance data generated by a representation function
$\bar{x}_i$ is the row data
n is the number of the creep compliance data

The creep compliance curves were generated by the three representation function used to fit the raw data at 0, 100, 150, 200, 250, and 300 F-T cycles and their comparisons were shown in Figure 5, Figure 7, Figure 9, Figure 11, Figure 13, and Figure 15 while the RMSE analyses using Eq. 13 at the selected F-T cycles were displayed in Figure 6, Figure 8, Figure 10, Figure 12, Figure 14, and Figure 16. The summed values of RMSE of each fit model were depicted in Table 1.

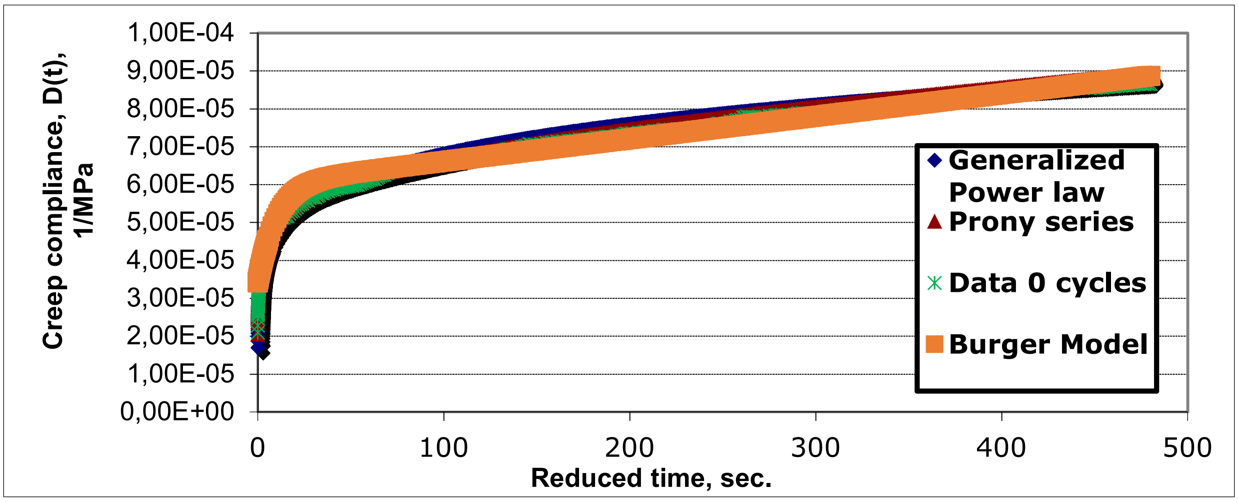

Figure 5: Comparison of representation functions in fitting raw data at 0 F-T cycle

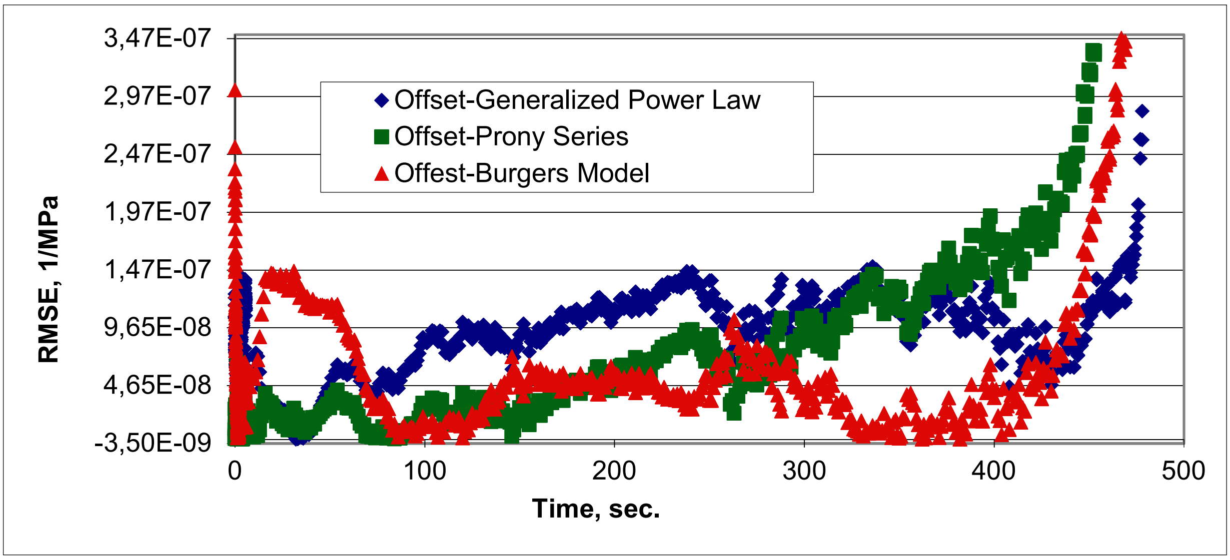

Figure 6: Comparison of RMSE at 0 F-T cycle

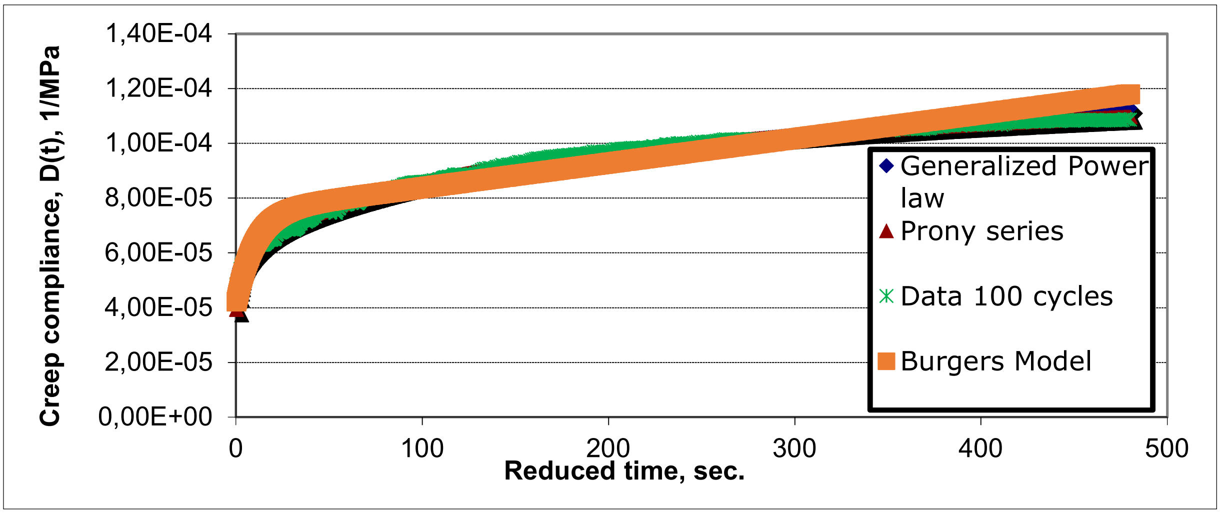

Figure 1: Comparison of representation functions in fitting raw data at 100 F-T cycle

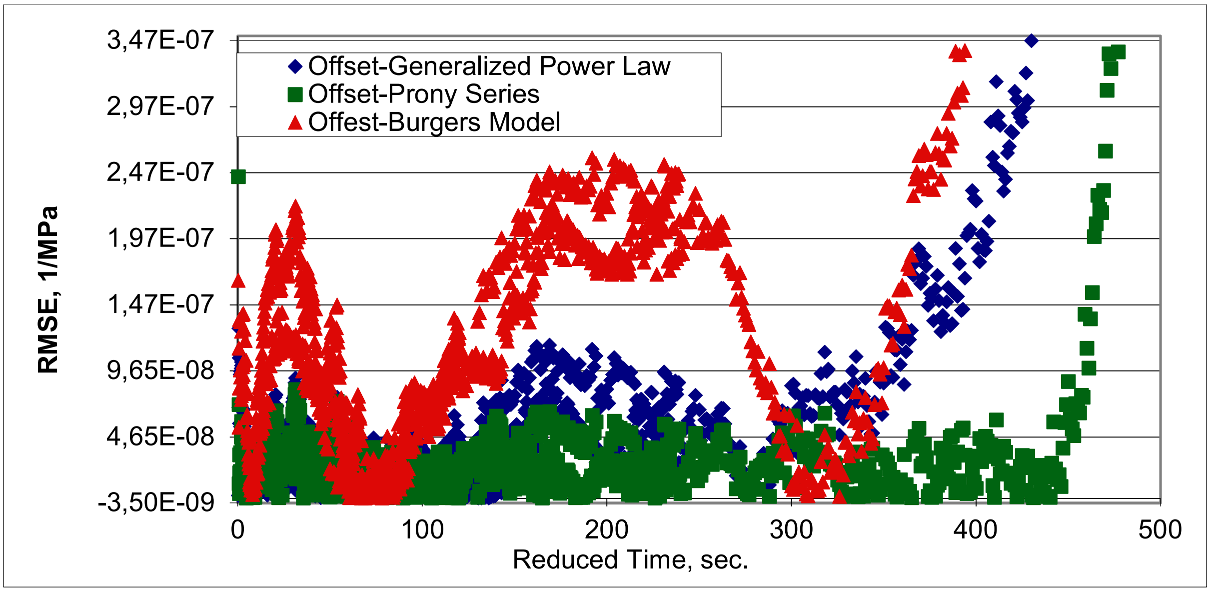

Figure 8: Comparison of RMSE at 100 F-T cycle

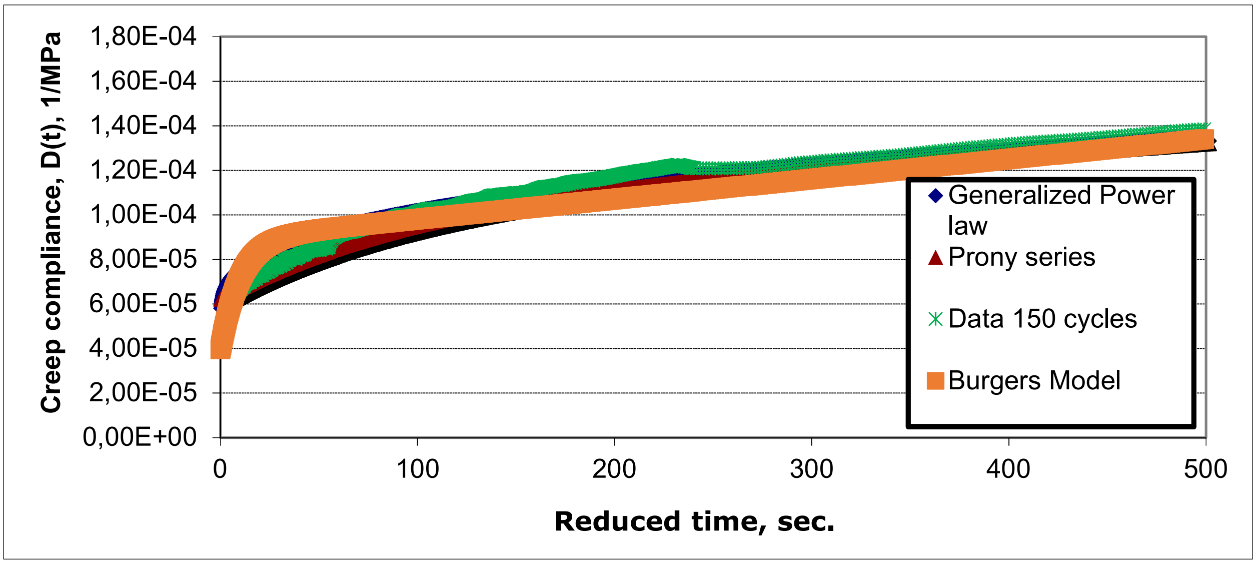

Figure 9: Comparison of representation functions in fitting raw data at 150 F-T cycle

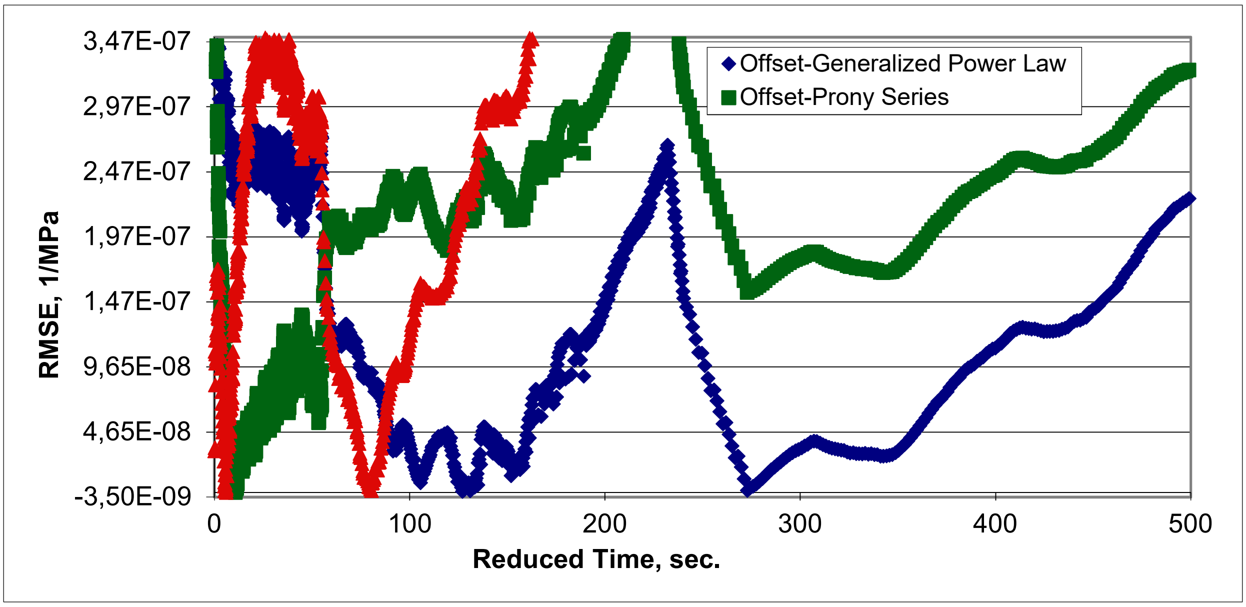

Figure 10: Comparison of RMSE at 150 F-T cycle

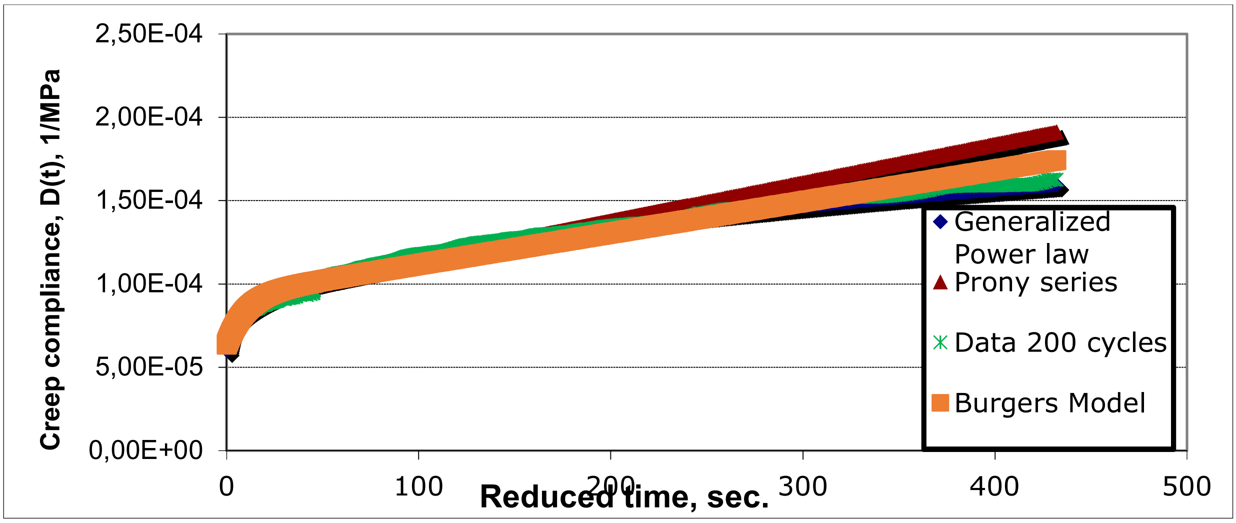

Figure 112: Comparison of representation functions in fitting raw data at 200 F-T cycle

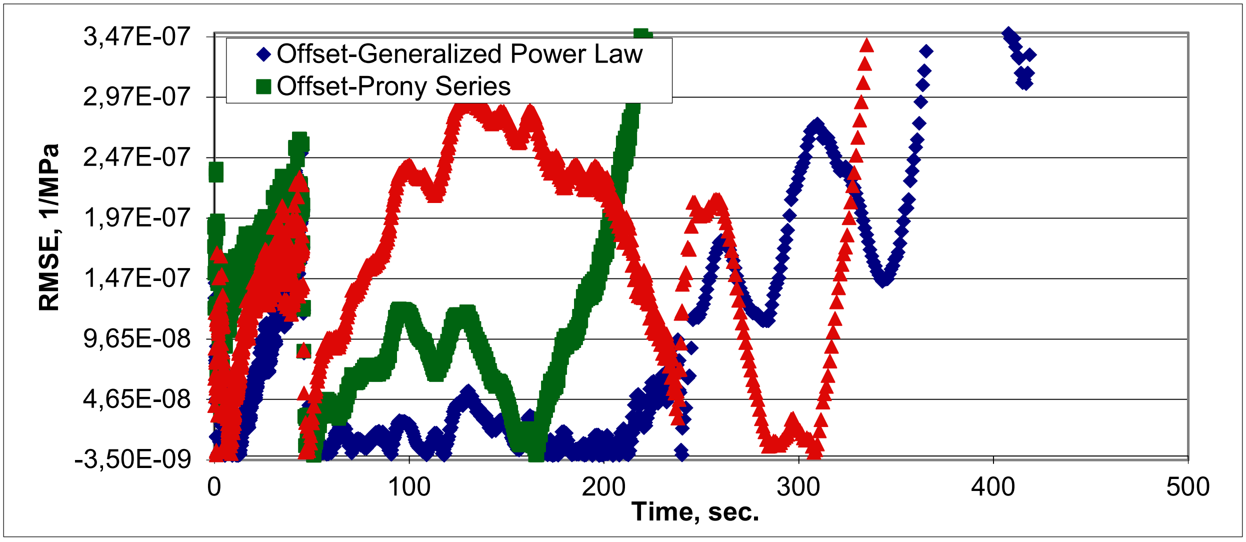

Figure 123: Comparison of RMSE at 200 F-T cycle

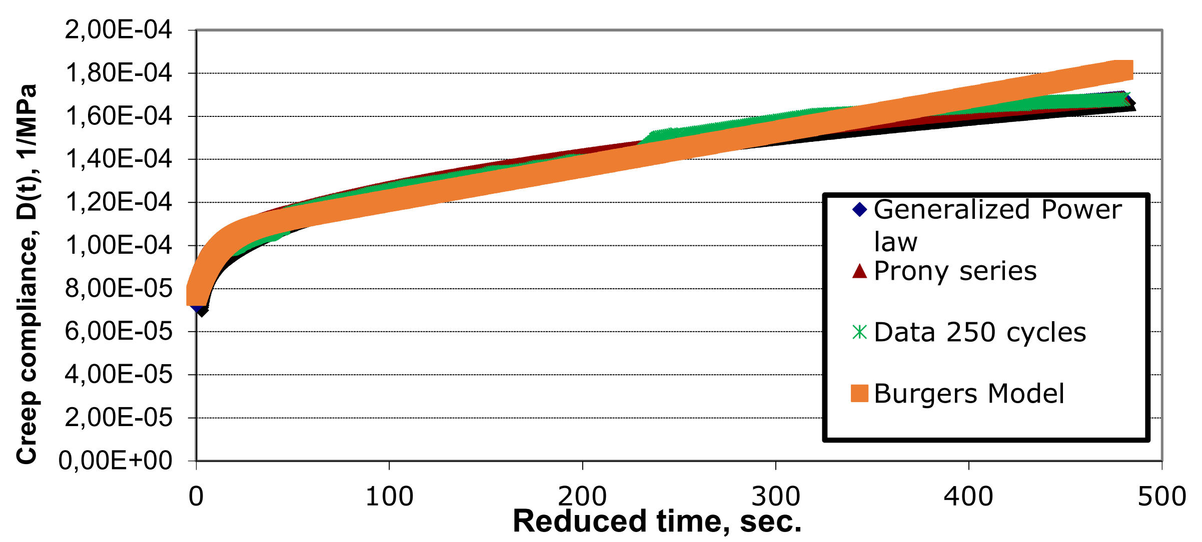

Figure 134: Comparison of representation functions in fitting raw data at 250 F-T cycle

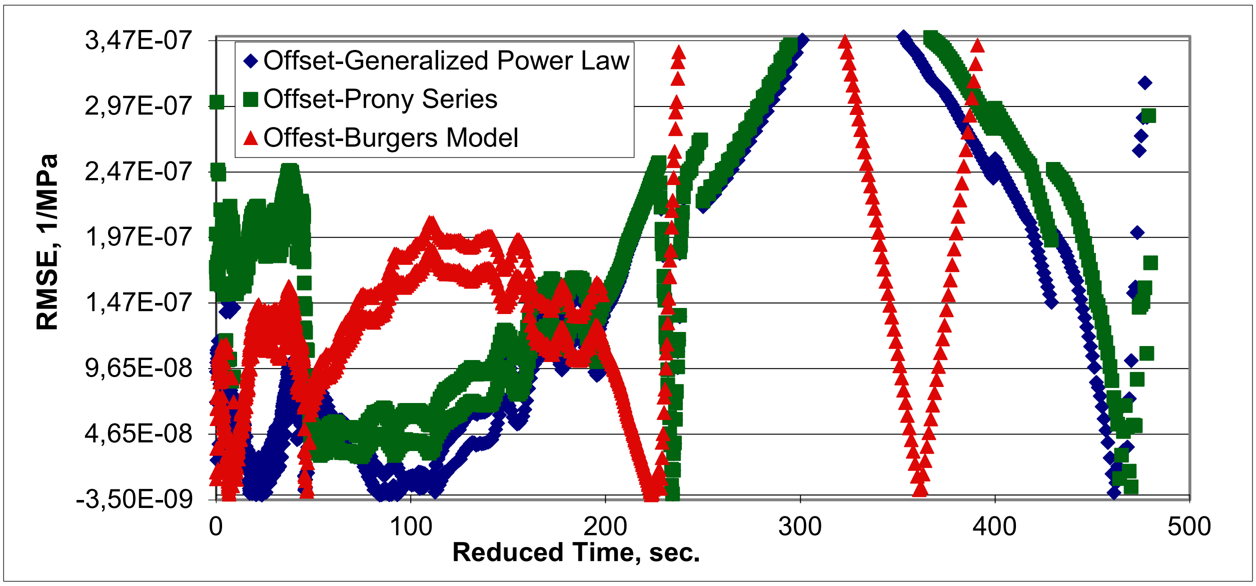

Figure 145: Comparison of RMSE at 250 F-T cycle

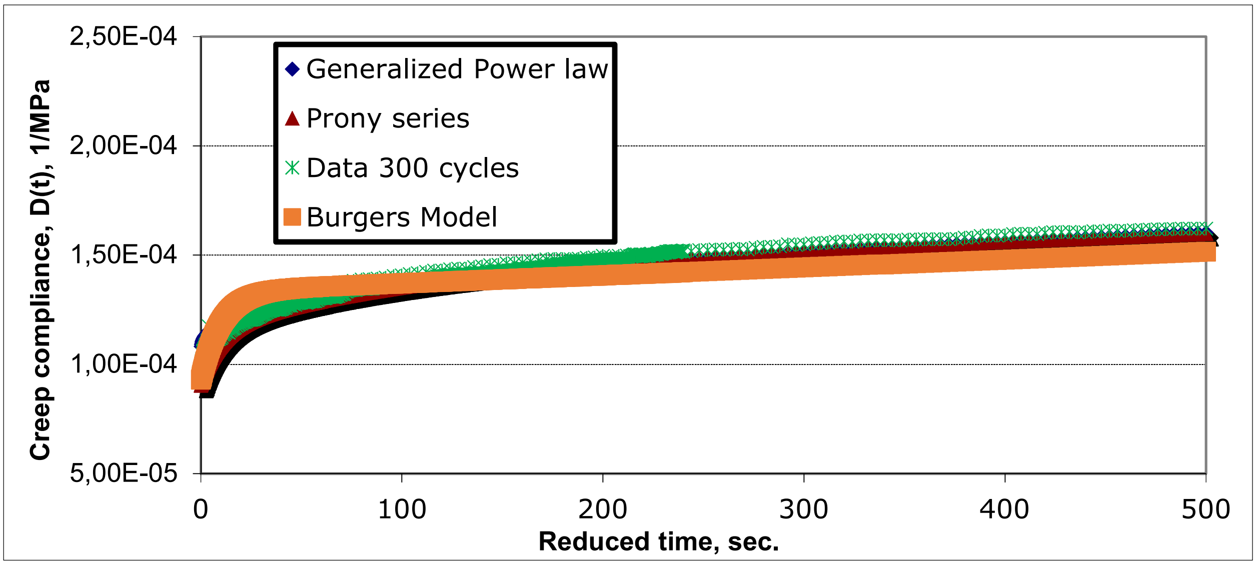

Figure 15: Comparison of representation functions in fitting raw data at 300 F-T cycle

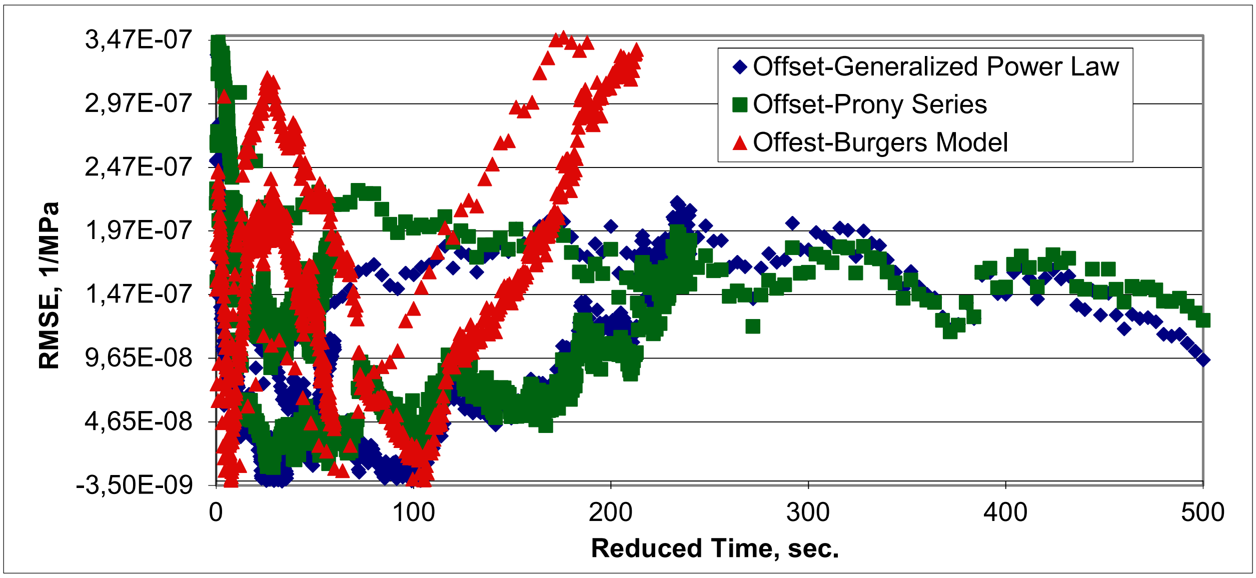

Figure 16: Comparison of RMSE at 300 F-T cycle

Table 1: Sum of root mean squared error (RMSE) of fitting results

	RMSE					
Model	0 cycle	100 cycles	150 cycles	200 cycles	250 cycles	300 cycles
Generalized Power Law	2.51E-06	1.33E-06	6.21E-06	2.33E-06	2.33E-06	2.64E-06
Prony Series	1.32E-06	8.17E-07	4.68E-06	8.02E-06	4.87E-06	4.08E-06
Burgers Model	1.83E-06	3.46E-06	7.83E-06	4.69E-06	4.11E-06	6.39E-06

Discussions

Based on the above comparisons (Figure 5 through Figure 16) and RMSE values in Table 1 among the three representation functions, Prony series function and Burger model seemed to be good options to fit raw creep compliance data. However, RMSE values shown in Table 1 do not show significant differences at all selected F-T cycles among the three representation functions. On the other hand, the three representation functions can be considered as a good fit to the braw creep compliance data, provided their RMSE values are very close to 0.

Conclusions

The analysis results conclude the Prony series and the BMBR function show good agreement with the BBR test results and both have better accuracy in predicting nonlinear responses of asphalt mixtures. The results presented in the paper can also be used to compare the prediction accuracy of three representation functions on nonlinear behaviors of asphalt mix that would benefit for future/further research.

Reference

[1] Christensen, D. W. and Anderson, D. A. 1992. Interpretation of Dynamic Mechanical Test Data for Paving Grade Asphalt Cements. Journal of the Association of Asphalt Paving Technologist: 67-116.

[2] Findley, W. N., Lai, J. S., and Onaran, K. 1989. Creep and relaxation of nonlinear viscoelastic materials. Dover Publications Inc., New York.

[3] Kim, Y.R. and Little, D. N. 2004. Linear Viscoelastic Analysis of Asphalt Mastics. Journal of Materials in Civil Engineering:16(2)122-132.

[4] Kim, J. S., Scholar, G. A., and Kim, S. G. 2008. Determination of Accurate Creep Compliance and Relaxation Modulus at a Single Temperature for Viscoelastic Solids. Journal of Materials in Civil Engineering: 20(2)147-156.

[5] Kim, J. S., Roque, R., and Byron, T. 2009. Viscoelastic Analysis of Flexible Pavements and its Effects on Top-Down Cracking. Journal of Materials in Civil Engineering: 21(7)324-332.

[6] Malkin, A. Y., and Masalova, I. 2001. From Dynamic Modulus via Different Relaxation Spectra to Relaxation and Creep Functions. Journal of Rheological Acta: (40)261-271.

[7] Marasteanu, M. O., and Anderson, D. A. 2000. Establishing linear viscoelastic conditions for asphalt binders. Journal of Transportation Research Record: (1728)1–6.

[8] Nikonov, A., Davies, A.R., and Emri, I. 2005. The determination of creep and relaxation modulus functions from a single experiment. Journal of Rheology: 49(6)1193-1211.

[9] Christensen, D. W. 1998. Analysis of Creep data from Indirect Tension Test on Asphalt Concrete**.** Journal of the Association of Asphalt Paving Technologist: 458-492

[10] Ho, C.H., and Romero, P. 2012. An Alternative Function to Represent Relaxation Modulus of Viscoelastic Materials. Journal of Materials in Civil Engineering: 24(24)152-158.

[11] Ho, C.H., Shan, J., Wang, F, and Chen, Y. 2016. Evaluating the feasibility of freeze-thaw cycling test on durability of asphalt mixtures: a pilot study. Proceedings of the 4th International Chinese European Workshop on functional pavement design in Delft, the Netherlands, June 29-July 1, 2016.

[12] Christensen, D.W. & Anderson, D.A. 1992. Interpretation of dynamic mechanical test data for paving grade asphalt cements. Journal of the Association of Asphalt Paving Technologists. (61):67-116.