Modified statistical linearization for analysing chaotic parametric space of weak-noise excited Duffing oscillator

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Summary. A novel stochastic linearization approach is proposed for predicting periodic and/or chaotic response of Duffing oscillator subjected to sinusoidal and weak-noise excitations. The present approach is realized by developing a modified statistical linearization method and incorporating an identification index, which is an incremental time average of variance response. Based on the modified statistical linearization model, the mean equation retains the same dynamic behaviour of the Duffing oscillator, and the linearized variance equation plays a role to monitor the generation of random behaviour in chaotic response. The parametric space of nonlinear response estimated by the present method is validated by employing Ueda parametric space as well as Lyapunov exponent and Monte Carlo simulations.

Introduction

In engineering applications, statistical linearization with Gaussian density, among the approximation methods, has been the mostly employed for analyzing the response behavior of general non-linear stochastic systems. Gaussian linearization method has been extended to investigate the regular cyclostationary response of a Duffing oscillator subjected to both sinusoidal and white noise excitations [1]. For the stochastic Duffing oscillator, the long-time response can be in a regular or irregular motion. Recently, a manifold independent method of utilizing an invariant measure of density response through Frobenius-Perron operator has been proposed for investigating chaotic response [2]. Regarding stochastic formulation of Frobenius-Perron operator, an invariant measure as the time average of density response can be obtained by solving Fokker-Planck-Kolmogorov (FPK) equation [2-4]. In literature, the investigation of the chaotic response of the stochastic Duffing oscillator is relied on numerical solution of FPK equation. However, the numerical solution cannot afford parametric relations for interpreting the physical mechanism in noisy chaotic response. The development of an approximate analysis through density response is essential for investigating chaotic response.

Modified statistical linearization

For the Duffing oscillator subjected to both sinusoidal and white noise excitations, at first, the nonlinear function in spring force will be linearized by following statistical linearization approach. Then, the mean and covariance propagation equations are derived from the linearized dynamic equation. Since the linearization model leads to linear and nonlinear propagation equations in the mean and covariance responses, respectively, the chaotic behavior, which is due to nonlinear dynamics, will not retained in the statistical linearization model. For retaining the nonlinear behavior in the linearization model, a modified statistical linearization is proposed. Regarding statistical linearization of a second-order system, the equation of mean response $m_i(t)$, i = 1, 2, consists of the variance response $h_{ij}(t)$, i, j = 1, 2. If the stochastic fluctuation of Duffing oscillator is weakly distributed, one can ignore the variance of displacement $h_{11}(t)$, which is appeared in the equation of mean of displacement $m_1(t)$. Thus, the formulated equation of mean propagation will be exactly the same as the original dynamic equation of Duffing oscillator. As a result, the mean equation retains the same dynamic behavior of the Duffing oscillator, and the linearized variance equation plays a role to monitor the generation of random behavior in chaotic response.

Results

For obtaining an invariant measure to identify chaotic response, the time average of variance responses $\overline{h}_{11}$ instead of $h_{11}(t)$ will be employed. The $\overline{h}_{11}$ by Gaussian and uniform densities are simulated by selecting appropriate parameters and variables of oscillator for generating periodic and chaotic responses as well as the coexistence of periodic and chaotic response. The input excitations consist of sinusoidal excitation $f\cos(\omega t)$ and weak Gaussian white noise $w(t)$. The simulation parameters and variables of the Duffing oscillator is listed in Table. 1. By ignoring the initial non-stationary $h_{11}(t)$, the simulated results of $\overline{h}_{11}$ is shown in Fig. 1. Fig. 1 reveals that the scattering region is chaotic response and the regular region is periodic response, despite of the probability density in linearization. The simulated results are verified by employing Lyapunov exponent and Monte Carlo simulations. Here, it is noted that chaotic response is a highly nonstationary process and the $\overline{h}_{11}$ will fluctuated in time. Thus, for the clear classification of chaotic and regular responses, an identification index $\Delta\overline{h}_{11}$, which is defined as consecutive increment of time average of data record of $h_{11}(t)$, is proposed. The simulated $\Delta\bar{h}_{11}$ by utilizing $40\pi/\omega$ data record and under different initial conditions $m_i(0)$ are shown in Fig. 2. In some regions of Fig. 2, for all four different initial conditions, the existence of zero and non-zero scattering data reveals periodic and chaotic response, respectively. In other regions, the apperance of zero or non-zero scattering data will depend on the initial conditions. These regions reveal that both periodic and chaotic responses co-exist.The simulated results by the modified statistical linearization through $\Delta\bar{h}_{11}$ are compared with Ueda parametric space [5] as well as the results by Lyapunov exponent and Monte Carlo simulations as shown in Fig. 3. Fig. 3 validates the validity of utilizing present method for identifying regular periodic and irregular chaotic responses.

Conclusions

An innovative modified statistical linearization method is proposed for investigating the chaotic parametric space of Duffing oscillator subjected to both sinusoidal and weak-noise excitations. Based on the modified statistical linearization model, the mean equation retains the same dynamic behaviour of the Duffing oscillator, and the linearized variance equation plays a role to monitor the generation of random behaviour in chaotic response. By incorporating an incremental time average of variance response as an identification index, the periodic and chaotic responses as well as the coexistence of periodic and chaotic response can be predicted. The validity of present method for identifying the response behaviour of Duffing oscillator is validated by employing Ueda parametric space as well as Lyapunov exponent and Monte Carlo simulations.

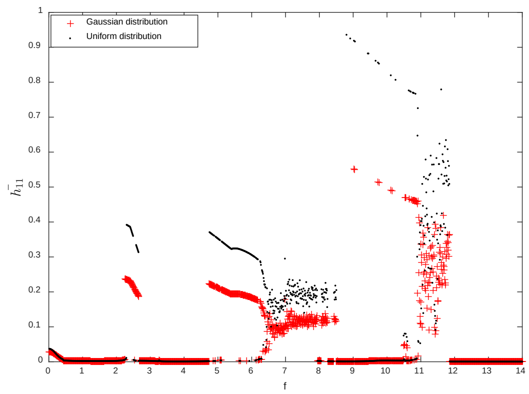

Fig. 1 Modified Gaussian and non-Gaussian linearization under initial zero mean value for obtaining time-average variance of x_1 with varying sinusoidal amplituide.

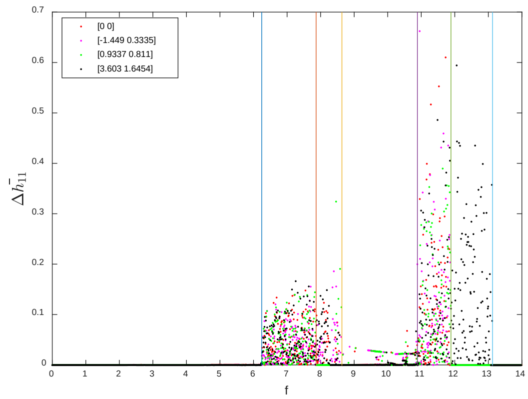

Fig. 2 Modified Gaussian linearization under different initial mean value, $[m_1(0)\ m_2(0)]$, for obtaining incremental time-average variance of x_1 with varying sinusoidal amplituide.

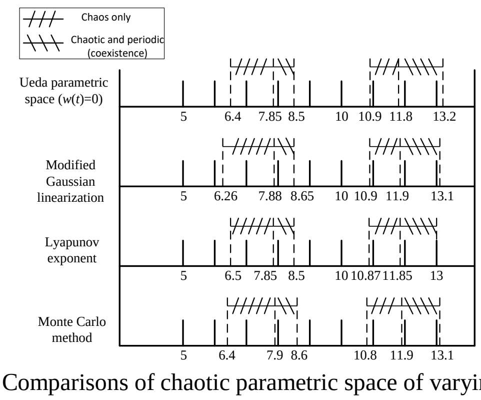

Fig. 3 Comparisons of chaotic parametric space of varying sinusoidal amplitude by different methods.

Table. 1 Simulation input for

$$\ddot{x} + \gamma\dot{x} + bx + cx^3 = f\cos(\omega t) + w(t)$$

Parameters and Variables	Value
γ	0.2
b	0
c	1
f	0~14
ω	1
Noise intensity of $w(t)$	0.001
$h_{11}(0)$, $h_{12}(0)$, $h_{22}(0)$	0.1, 0.1, 0.1

References

[1] Chang R. J. (2015) Cyclostationary Gaussian Linearization for Analysing Nonlinear System Response under Sinusoidal Signal and White Noise Excitation. *Int. J. Mech. Aero. Indus. Mechatr. Manuf. Eng*. 9(5):738-745.
[2] Naess, A. (2000) Chaos and Nonlinear Stochastic Dynamics. *Prob. Eng. Mech.*15:37-47.
[3] Kunert A. Pfeiffer F. (1991) Description of Chaotic Motion by an Invariant Probability Density. *Nonlinear Dyn.* 2:291-304.
[4] Kapitaniak, T. (1990) Chaos in Systems with Noise, 2nd. World Scientific, Singapore.
[5] Ueda, Y. (1980) Steady Motions Exhibited by Duffing's Equation: A Picture Book of Regular and Chaotic Motions, in New Approaches to Nonlinear Problems in Dynamics, ed. P. J. Holmes, SIAM, Pennsylvania, USA.